DRAFT

Content Specifications
for the Summative assessment of the
Common Core State Standards for Mathematics

REVIEW DRAFT

Available for Consortium and Stakeholder Review and Feedback
December 9, 2011

Developed with input from content experts and SMARTER Balanced Assessment Consortium Staff, Work Group Members, and Technical Advisory Committee

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An SMARTER Balanced-focused version of the Working Group report may be found at http://www.mathshell.org/papers/pdf/ISDDE_SBAC_Feb11.pdf

Table of Contents

Introduction and Background

Using This Document: This version of the SMARTER Balanced Assessment Consortium's work on Content Specifications and Content Mapping is presented as a set of several materials and includes changes based on the productive feedback provided to the consortium following the first round of commentary. This version, the second of two public releases available for review and feedback, invites commentary from all interested stakeholders in the Consortium's work. Instructions on how to submit comments and feedback can be found in the Resources section of the Consortium's Web site: www.smarterbalanced.org

Pages 1-120 represent the core of this document, and should be read carefully for comment and feedback. Three sets of appendices are intended to provide further elaboration of our work so far. All three sets – Appendix A, B and C – are embedded in this document, as it might be most useful for a reader to have them ready at hand. The last set – Appendix C – provides examples of items and tasks that illustrate the intent of the content standards.

In addition to this document, we are again making available an online survey for stakeholder feedback. We know there is a lot of interest in this release, and anticipate a very large volume of feedback. To ensure that comments and suggestions are received and considered, **we ask readers to be sure to use the online survey** as the vehicle for providing responses.

This document follows an earlier release by the Consortium of a companion document covering specifications for English language arts and literacy. These documents seek comment from Consortium members and other stakeholders. The table below outlines the schedule for the two rounds of public review for the content specifications of mathematics and English language arts/literacy.

SMARTER Balanced Content Specifications Development Timelines and Activities

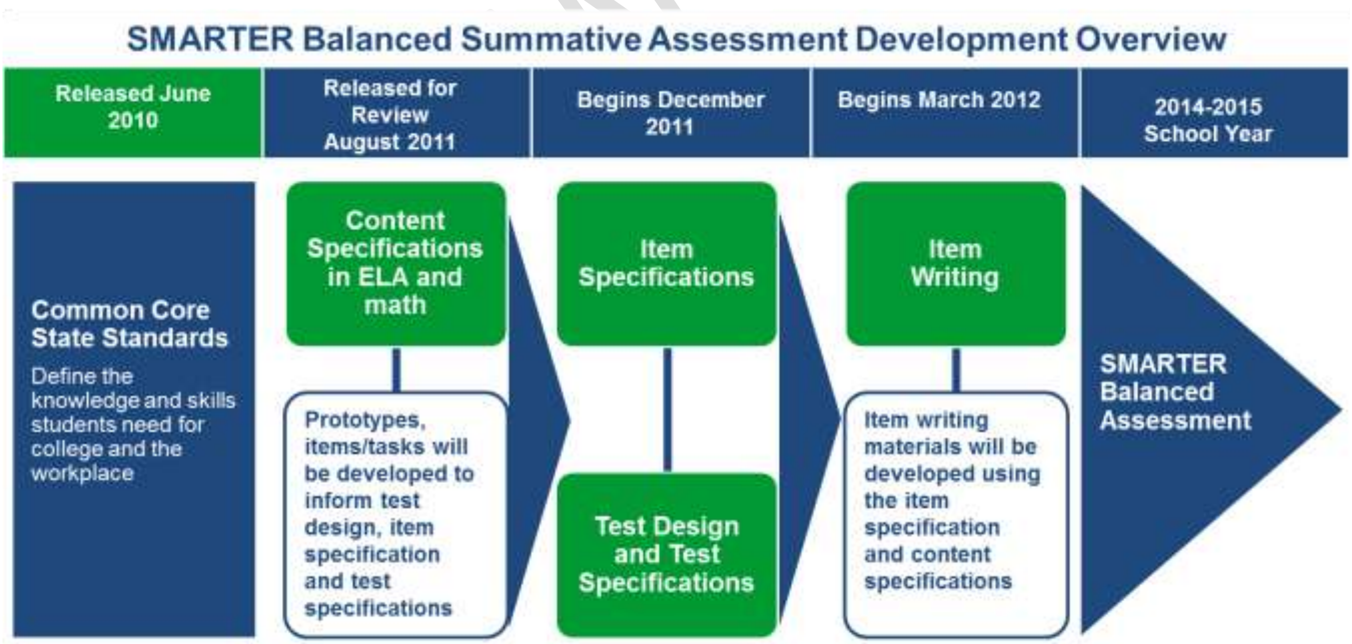
Review Steps	Date
Internal Review Start: ELA/Literacy - ELA/Literacy content specifications distributed to specific SMARTER Balanced work groups for initial review and feedback	07/05 (Tue)
Internal Review Due: ELA/Literacy - Emailed to SMARTER Balanced	07/15 (Fri)
Technical Advisory Committee (TAC) Review Liaison Review: ELA/Literacy - Draft submitted to TAC for review, comment, and feedback	07/27 (Wed)
Webinar: ELA/Literacy (including Evidence Based Design orientation) - Orientation for SMARTER Balanced members to Evidence Based Design and walkthrough of draft ELA/Literacy specifications document	08/08 (Mon)
Release for Review: ELA/Literacy (Round 1) - ELA/Literacy specifications documents posted to www.smarterbalanced.org and emailed to stakeholder groups	08/09 (Tue)
Internal Review Start: Mathematics - Mathematics content specifications distributed to specific SMARTER Balanced work groups for preliminary review and feedback	08/10 (Wed)

Technical Advisory Committee (TAC) Review Liaison Review: Mathematics - Draft submitted to TAC for review, comment, and feedback	08/10 (Wed)
Internal Review Due: Mathematics - Emailed to SMARTER Balanced	08/15 (Mon)
Release to Item Specifications to Bidders: ELA/Literacy - Current drafts of ELA/Literacy content specifications posted to www.smarterbalanced.org to support Item Specifications RFP process	08/15 (Mon)
Webinar: Mathematics - Walkthrough for SMARTER Balanced members of the draft Mathematics specifications document	08/29 (Mon)
Release for Review: Mathematics (Round 1) - Mathematics content specifications posted www.smarterbalanced.org and emailed to stakeholder groups	08/29 (Mon)
Release of Specifications to Bidders: Mathematics - Current drafts of Mathematics content specifications posted to www.smarterbalanced.org to support Item Specifications RFP process	08/29 (Mon)
Feedback Surveys Due: ELA/Literacy (Round 1) - Emailed to SMARTER Balanced	08/29 (Mon)
Feedback Surveys Due: Mathematics (Round 1) - Emailed to SMARTER Balanced	09/19 (Mon)
Release for Review: ELA/Literacy (Round 2) - ELA content specifications posted to www.smarterbalanced.org and emailed to stakeholder groups	09/20 (Tue)
Feedback Surveys Due: ELA/Literacy (Round 2) - Emailed to SMARTER Balanced	09/27 (Tue)
Release for Review: Mathematics (Round 2) - Mathematics content specifications posted to www.smarterbalanced.org; email notification sent to stakeholder groups	12/09 (Fri)
Feedback Surveys Due: Mathematics (Round 2) - Emailed to SMARTER Balanced	01/03/12 (Tue)
ELA/Literacy Claims Webinar Discussion - Summative assessment claims are discussed in preparation for subsequent vote by Governing states. Voting will be open 1/11/12 through 1/18/12.	01/10/12 (Tue)
Mathematics Claims Webinar Discussion - Summative assessment claims are discussed in preparation for subsequent vote by Governing states. Voting will be open 1/24/12 through 2/1/12.	01/24/12 (Tue)
ELA/Literacy Claims adopted by Governing States - Summative assessment claims are established as policy for the Consortium through email voting of governing state chiefs.	Mid Jan 2012
Final Content Specifications and Content Mapping Released: ELA/Literacy - Final ELA/Literacy content specifications posted to www.smarterbalanced.org; email notification sent to member states and partner organizations	Late Jan 2012
Mathematics Claims adopted by Governing States - Summative assessment claims are established as policy for the Consortium through email voting of governing state chiefs.	Late Jan 2012
Final Content Specifications and Content Mapping Released: Mathematics - Final mathematics content specifications posted to www.smarterbalanced.org; email notification sent to member states and partner organizations	Early Feb 2012

The contents of this document describe the Consortium's current specification of critically important claims about student learning that are derived from the Common Core State Standards. When finalized, these claims will serve as the basis for the Consortium's system of summative and interim assessments and its formative assessment support for teachers. Open and transparent decision-making is one of the Consortium's central principles. This draft of the mathematics content specifications is being made available for comment consistent with that principle, and all responses to this work will be considered as it continues to be refined.

Purpose of the content specifications: The SMARTER Balanced Assessment Consortium is developing a comprehensive assessment system for mathematics and English language arts / literacy—aligned to the Common Core State Standards—with the goal of preparing all students for success in college and the workforce. Developed in partnership with member states, leading researchers, content expert experts, and the authors of the Common Core, content specifications are intended to ensure that the assessment system accurately assesses the full range the standards.

This content specification of the Common Core mathematics standards provides clear and rigorous focused assessment targets that will be used to translate the grade-level Common Core standards into content frameworks along a learning continuum, from which test blueprints and item/task specifications will be established. Assessment evidence at each grade level provides item and task specificity and clarifies the connections between instructional processes and assessment outcomes.



The Consortium Theory of Action for Assessment Systems: As stated in the SMARTER Balanced Assessment Consortium's (SMARTER Balanced) Race to the Top proposal, "the Consortium's Theory of Action calls for full integration of the learning and assessment systems, leading to more informed decision-making and higher-quality instruction, and ultimately to increased numbers of students who are well prepared for college and careers." (p. 31). To that end, SMARTER Balanced's proposed system features rigorous content standards; common adaptive summative assessments that make use of technology-enhanced item types, as well as extended performance tasks that provide students the opportunities to demonstrate proficiency both with content and in the mathematical practices described in the Common Core State Standards; computer adaptive interim assessments that provide mid-course information about what students know and can do; instructionally sensitive formative tools, processes, and practices that can be accessed on-demand; focused ongoing support to teachers through professional development opportunities and exemplary instructional materials; and an online, tailored, reporting and tracking system that allows teachers, administrators, and students to access information about progress towards achieving college- and career-readiness as well as to identify specific strengths and weaknesses along the way. Each of these components serve to support the Consortium's overarching goal: *to ensure that all students leave high school prepared for post-secondary success in college or a career through increased student learning and improved teaching*. Meeting this goal will require the coordination of many elements across the educational system, including but not limited to a quality assessment system that strategically "balances" summative, interim, and formative components (Darling-Hammond & Pecheone, 2010; SMARTER Balanced, 2010).

The proposed SMARTER Balanced mathematics assessments and the assessment system are shaped by a set of characteristics shared by the systems of high-achieving nations and states, and include the following principles:[1]

1) **Assessments are grounded in a thoughtful, standards-based curriculum and are managed as part of an <u>integrated system</u>** of standards, curriculum, assessment, instruction, and teacher development. Curriculum and assessments are organized around a set of learning progressions[2] along multiple dimensions within subject areas. These guide teaching decisions, classroom-based assessment, and external assessment.

2) **Assessments include <u>evidence of student performance</u>** on challenging tasks that evaluate Common Core Standards of 21st century learning. Instruction and assessments seek to teach and evaluate knowledge and skills that generalize and can transfer to higher education and multiple work domains. They emphasize deep knowledge of core concepts and ideas within and across the disciplines, along with analysis, synthesis, problem solving, communication, and critical thinking. This kind of learning and teaching requires a focus on complex performances as well as the testing of specific concepts, facts, and skills.

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[1] Darling-Hammond, L. (2010) <u>Performance counts</u>. Washington, DC: Council of Chief State School Officers.

[2] Empirically-based learning progressions visually and verbally articulate a hypothesis, or an anticipated path, of how student learning will typically move toward increased understanding over time with good instruction (Hess, Kurizaki, & Holt, 2009). The major concept of learning progressions is that students should progress through mathematics by building on what they know, moving toward some defined goals. While the structure of the mathematics shapes the pathways, there is *not* one prescribed or optimal pathway through the content.

3) **<u>Teachers are integrally involved</u> in the development and scoring of assessments**. While many assessment components can and will be efficiently and effectively scored with computer assistance, teachers will also be involved in the interim/benchmark, formative, and summative assessment systems so that they deeply understand and can teach to the standards.

4) **Assessments are structured to <u>continuously improve teaching and learning.</u>** Assessment *as*, *of*, and *for* learning is designed to develop understanding of what learning standards are, what high-quality work looks like, what growth is occurring, and what is needed for student learning. This includes:

    - <u>Developing assessments around learning progressions</u> that allow teachers to see what students know and can do on multiple dimensions of learning and to strategically support their progress;
    - <u>Using computer-based technologies to adapt assessments to student levels</u> to more effectively measure what they know, so that teachers can target instruction more carefully and can evaluate growth over time;
    - <u>Creating opportunities for students and teachers to get feedback on student learning throughout the school year</u>, in forms that are actionable for improving success;
    - <u>Providing curriculum-embedded assessments that offer models of good curriculum and assessment practice,</u> enhance curriculum equity within and across schools, and allow teachers to see and evaluate student learning in ways that can feed back into instructional and curriculum decisions; and
    - <u>Allowing close examination of student work</u> and moderated teacher scoring as sources of ongoing professional development.

5) **Assessment, reporting, and accountability systems provide <u>useful information on multiple measures</u> that is educative for all stakeholders.** Reporting of assessment results is timely, specific, and vivid—offering specific information about areas of performance and examples of student responses along with illustrative benchmarks, so that teachers and students can follow up with targeted instruction. Multiple assessment opportunities (formative and interim/benchmark, as well as summative) offer ongoing information about learning and improvement. Reports to stakeholders beyond the school provide specific data, examples, and illustrations so that administrators and policymakers can more fully understand what students know in order to guide curriculum and professional development decisions.

Accessibility to Content Standards and Assessments: In addition to these five principles, SMARTER Balanced is committed to ensuring that the content standards, summative assessments, teacher-developed performance tasks, and interim assessments adhere to the principles of accessibility for students with disabilities and English Language Learners.[3] It is important to understand that the purpose

[3] Accessibility in assessments refers to moving "beyond merely providing a way for students to participate in assessments. Accessible assessments provide a means for determining whether the knowledge and skills of each student meet standards-

of accessibility is ***not*** to reduce the rigor of the Common Core State Standards, but rather to avoid the creation of barriers for students who may need to demonstrate their knowledge and skills at the same level of rigor in different ways. Toward this end, each of the claims for the CCSS in Mathematics is briefly clarified in terms of accessibility considerations. Information on what this means for content specifications and mapping will be developed further during the test and item development phases.

Too often, individuals knowledgeable about students with disabilities and English learners are not included at the beginning of the process of thinking about standards and assessments, with the result being that artificial barriers are set up in the definition of the content domain and the specification of how the content maps onto the assessment. These barriers can prevent these students from showing their knowledge and skills via assessments. The focus on "accessibility," as well as the five principles shared by systems of high-achieving nations and states, underlies the Consortium's approach to content mapping and the development of content specifications for the SMARTER Balanced assessment system.

Accessibility is a broad term that covers both instruction (including access to the general education curriculum) and assessment (including summative, interim, and formative assessment tools). *Universal design* is another term that has been used to convey this approach to instruction and assessment (Johnstone, Thompson, Miller, & Thurlow, 2008; Rose, Meyer, & Hitchcock, 2005; Thompson, Thurlow, & Malouf, 2004; Thurlow, Johnstone, & Ketterline Geller, 2008; Thurlow, Johnstone, Thompson, & Case, 2008). The primary goal is to move beyond merely *including* students in instruction or assessment, but to (a) to ensure that students learn what other students learn, and (b) to determine whether the knowledge and skills of each student meet standards-based criteria.

Several approaches have been developed to meet the two major goals of accessibility and universal design. They include a focus on multiple means of representation, multiple means of expression, and multiple means of engagement for instruction. Use of multiple media is also a key feature of accessibility. Elements of universally designed assessments and considerations for item and test review are a focus for developing accessible assessments. Increased attention has been given to computer-based assessments (Thurlow, Lazarus, Albus, & Hodgson, 2010) and the need to establish common protocols for item and test development, such as those described by Mattson and Russell (2010).

For assessments, the goal for all students with disabilities (except those students with significant cognitive disabilities who participate in an alternate assessment based on alternate achievement standards) is to measure the same knowledge and skills at the same level as traditional assessments, be they summative, interim, or formative assessments. Accessibility does not entail measuring different knowledge and skills for students with disabilities from what would be measured for peers without

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based criteria. This is not to say that accessible assessments are designed to measure whatever knowledge and skills a student happens to have. Rather, they measure the same knowledge and skills at the same level as traditional … assessments. Accessibility does not entail measuring different knowledge and skills for students with disabilities [or English Language Learners] from what would be measured for peers without disabilities" (Thurlow, Laitusis, Dillon, Cook, Moen, Abedi, & O'Brien, 2009, p. 2).

disabilities (Thurlow, Laitusis, Dillon, Cook, Moen, Abedi, & O'Brien, 2009; Thurlow, Quenemoen, Lazarus, Moen, Johnstone, Liu, Christensen, Albus, & Altman, 2008). It does entail understanding the characteristics and needs of students with disabilities and addressing ways to design assessments and provide accommodations to get around the barriers created by their disabilities.

Similarly, the goal for students who are English language learners is to ensure that performance is not impeded by the use of language that creates barriers that are unrelated to the construct being measured. Unnecessary linguistic complexity may affect the accessibility of assessments for all students, particularly for those who are non-native speakers of English (Abedi, in press; Abedi, 2010; Solano-Flores, 2008). Several studies have shown how the performance of ELL students can be confounded during mathematics assessments as a function of unfamiliar cultural referents and unnecessary linguistic complexities (see for example, Abedi, 2010; Abedi & Lord, 2001; Solano-Flores, 2008).

In particular, research has demonstrated that several linguistic features unrelated to mathematics content could slow the reader down, increase the possibility of misinterpretation of mathematics items, and add to the ELL student's cognitive load, thus interfering with understanding the assessment questions and explaining the outcomes of assessments. Indices of language difficulty that may be unrelated to the mathematics content include unfamiliar (or less commonly used) vocabulary, complex grammatical structures, and styles of discourse that include extra material, conditional clauses, abstractions, and passive voice construction (Abedi, 2010a).

A distinction has been made between language that is relevant to the focal construct (mathematics in this case) and language that is irrelevant to the content (construct-irrelevant). SMARTER Balanced intends to address issues concerning the impact of unnecessary linguistic complexity of mathematics items as a source of construct-irrelevant factor for ELL students, and provide guidelines on how to control for such sources of threat to the reliability and validity of mathematics assessments for these students. Studies on the impact of language factors on the assessment outcomes have also demonstrated that they impact performance of students with learning and reading disabilities. Thus, controlling for such sources of impact will also help students with learning/reading disabilities (Abedi, 2010b).

In addition, ELL students' abilities to communicate could substantially confound their level of proficiency in mathematics, as it is required for many of the mathematical tasks. For example, a major requirement for a successful performance in mathematics as outlined in the CCSSM is a high level of verbal and written communication skills. Each of the four claims indicates that successful completion of mathematics operations may not be sufficient to claim success in the tasks and that students should also be able to clearly and fluently communicate their reasoning. This could be a major obstacle for ELL students who are highly proficient in mathematical concepts and mathematical operations but not at the level of proficiency in English to provide clear explanation of the operations in words alone. Allowing students to show their reasoning using mathematical models, equations, diagrams, and drawings as well as written text will provide more complete access to students' thinking and understanding.

In the case of English learners (EL), ensuring appropriate assessment will require a reliable and valid measure of EL students' level of proficiency in their native language (L1) and in English (L2). In general, if students are not proficient in English but are proficient in L1 and have been instructed in L1, then a native language version of the assessment should be considered, since an English version of the assessment will not provide a reliable and valid measure of students' abilities to read, write, listen, and speak. If students are at the level of proficiency in reading in English to meaningfully participate in an English-only assessment (based, for example, on a screening test or the Title III ELP assessment), then it will be appropriate to provide access in a computer adaptive mode to items that are consistent with their level of English proficiency but measure the same construct as other items in the pool. (See Abedi, et al 2011 for a computer adaptive system based on students' level of English language proficiency.)

As issues of accessibility are being considered, attention first should be given to ensuring that the design of the assessment itself does not create barriers that interfere with students showing what they know and can do in relation to the content standards. Several approaches to doing this were used in the development of alternate assessments based on modified achievement standards and could be brought into regular assessments to meet the needs of all students, not just those with disabilities, once the content is more carefully defined. To determine whether a complex linguistic structure in the assessment is a necessary part of the construct (i.e., construct-relevant), a group of experts (including content and linguistic experts and teachers) should convene at the test development phase and determine all the construct-relevant language in the assessments. This analysis is part of the universal design process.

Accommodations then should be identified that will provide access for students who still need assistance getting around the barriers created by their disabilities or their level of English language proficiency after the assessments themselves are as accessible as possible. For example, where it is appropriate, items may be prepared at different levels of linguistic complexity so that students can have the opportunity to respond to the items that are more relevant for them based on their needs, ensuring that the focal constructs are not altered when making assessments more linguistically accessible. Both approaches (designing accessible assessments and identifying appropriate accommodations) require careful definition of the content to be assessed.

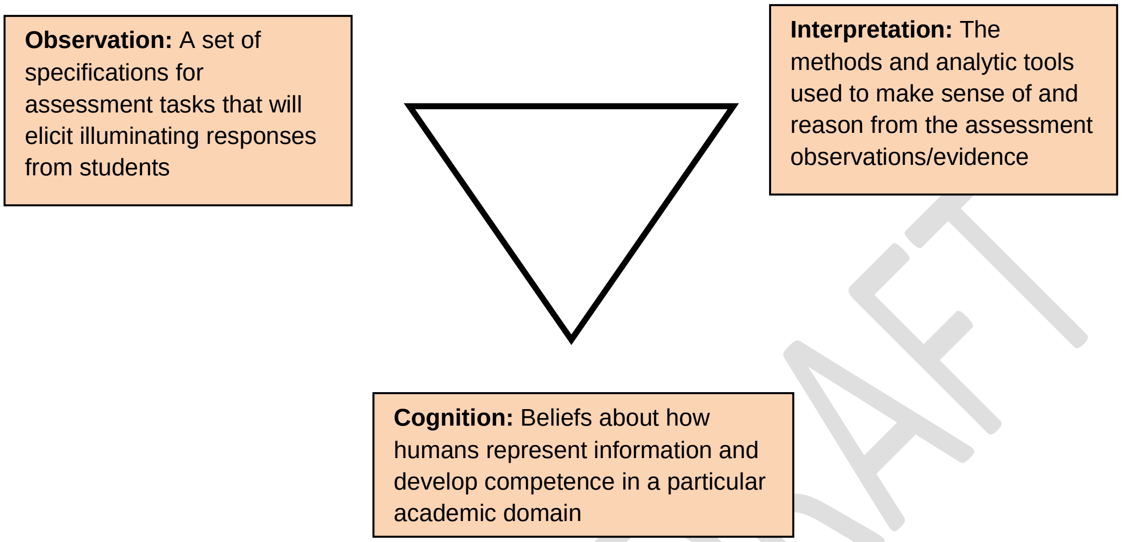
Careful definitions of the content are being created by SMARTER Balanced. These definitions involve identifying the SMARTER Balanced assessment claims, the rationale for them, what sufficient evidence looks like, and possible reporting categories for each claim. Further explication of these claims provides the basis for ensuring the accessibility of the content – accessibility that does not compromise the intended content for instruction and assessment – as well as accommodations that might be used without changing the content. Sample explications are provided under each of the claims.

Further Readings: Each of the SBAC assessment system principles is interwoven throughout this document in describing the content mapping and content specifications. Readers may want to engage in additional background reading to better understand how the concepts below have influenced the development of the SBAC mathematics assessment design.

- **Principles of evidence-based design (EBD); The Assessment Triangle (see next page); Cognition and transfer; Performances of novices/experts**
  (see Pellegrino, Chudowsky, & Glaser, 2001; Pellegrino, 2002)
- **Enduring understandings, transfer**
  (see Wiggins & McTighe, 2001)
- **Principles of evidence-centered design (ECD) for assessment**
  (see Mislevy, 1993, 1995)
- **Learning progressions/learning progressions frameworks**
  (see Hess, 2008, 2010, 2011; National Assessment Governing Board, 2007; Popham, 2011; Wilson, 2009)
- **Universal Design for Learning (UDL); Increased accessibility of test items**
  (see Abedi**,** 2010; Bechard, Russell, Camacho, Thurlow, Ketterlin Geller, Godin, McDivitt, Hess, & Cameto, 2009; Hess, McDivitt, & Fincher, 2008).
- **Cognitive rigor, Depth of Knowledge; Deep learning**
  (see Alliance for Excellence in Education, 2011; Hess, Carlock, Jones, & Walkup, 2009; Webb, 1999)
- **Interim assessment; Formative Assessment**
  (see Perie, Marion, & Gong, 2007; Heritage, 2010; Popham, 2011; Wiliam, 2011)
- **Constructing Questions and Tasks for Technology Platforms**
  (see Scalise & Gifford, 2006)

Content Mapping and Content Specifications for Assessment Design: The Assessment Triangle, illustrated on the following page, was first presented by Pellegrino, Chudowsky, and Glaser in *Knowing What Students Know/*KWSK (NRC, 2001.) "[T]he corners of the triangle represent the three key elements underlying any assessment…a model of student ***cognition*** and learning in the domain, a set of beliefs about the kinds of ***observations*** that will provide evidence of students' competencies, and an ***interpretation*** process for making sense of the evidence" (NRC, 2001, p. 44). KWSK uses the heuristic of this 'assessment triangle' to illustrate the fundamental components of evidence-based design (EBD), which articulates the relationships among learning models (Cognition), assessment methods (Observation), and inferences one can draw from the observations made about what students truly know and can do (Interpretation) (Hess, Burdge, & Clayton, 2011).

Application of the assessment triangle not only contributes to better test design. The interconnections among Cognition, Observation, and Interpretation can be used to gain insights into student learning. For example, learning progressions offer a coherent starting point for thinking about how students develop competence in an academic domain and how to observe and interpret the learning as it unfolds over time. These hypotheses about typical pathways of learning can be validated, in part, through systematic (empirical) observation methods and analyses of evidence produced in student work samples from a range of assessments.



The Assessment Triangle (NRC, 2001, p. 44)

Evidence-based design: SMARTER Balanced is committed to using evidence-based design in its development of assessments in the Consortium's system. The SMARTER Balanced approach is detailed in the following section, but a brief explanation is as follows. In this document, four "claims" are set forth regarding what students should know and be able to do in the domain of mathematics. Each claim is accompanied by a "Rationale" that provides the basis for establishing the claim as central to mathematics. The claims and Rationales represent the "cognition" part of the assessment triangle. For each claim and Rationale there is a section representing the "observation" corner of the triangle. Here, a narrative description lays out the kinds of evidence that would be sufficient to support the claim, which is followed by tables with "Assessment Targets" linked to the Common Core standards. Finally, the "interpretation" corner of the triangle is represented by a section for each claim that lists the "Proposed Reporting Categories" that the assessment would provide.

Part I – General Considerations for the Use of Items and Tasks to Assess Mathematics Content and Practice

Assessing Mathematics: The Common Core State Standards for mathematics require that mathematical content and mathematical practices be *connected* (CCSSM, p. 8). In addition, two of the major design principles of the standards are *focus* and *coherence* (CCSSM, p. 3). Together, these features of the standards have important implications for the design of the SMARTER Balanced assessment system.

Using Various Types of Items and Tasks to Connect Content and Practice: There are multiple dimensions to mathematical proficiency, ranging from knowing important mathematical facts and procedures to being able to use that knowledge in the solution of complex problems. SMARTER Balanced intends to use a variety of types of assessment items and tasks to assess student mathematical proficiency. For example, knowledge of how to add fractions, or how to solve two linear equations in two unknowns can be assessed with selected response or completion items. However, demonstrating the skills to model a mathematical situation and explain the rationale for the approach depends on deciding what is mathematically important in that situation, representing it with mathematical symbolism, operating on the symbols appropriately, and then interpreting the results in meaningful ways. Assessing this deeper understanding of mathematics can best be accomplished through the use of more complex assessment tasks. As elaborated below, a balanced and meaningful assessment should contain a spectrum of items and tasks, ranging from brief items targeting particular concepts or skills through more elaborate constructed response items and tasks that call upon the application of mathematical concepts to complex real-world scenarios that require students to make connections among multiple practices as described in CCSSM.

Focus and Coherence: The principles of focus and coherence on which the CCSSM are based have additional implications for mathematics assessment and instruction. Coherence implies that the standards are more than a mere checklist of disconnected statements; the cluster headings, domains, and other text in the standards all organize the content in ways that highlight the unity of the subject. The standards' focus is meant to allow time for students and teachers to master the intricate, challenging, and necessary things in each grade that open the way to a variety of applications even as they form the prerequisite study for future grades' learning. A quality assessment should strive to reinforce focus and coherence at each grade level by testing for proficiency with central and pivotal mathematics rather than covering too many ideas superficially – a key point of the Common Core Standards.

Judicious Coverage of the Standards: CCSSM describes a body of mathematical content and practices that students are to learn. Thus, an assessment aligned to the CCSSM must be faithful to that description of the mathematics, taken as a whole, and not simply to individual standards. Such assessments can be constructed by a) beginning with the selection of complex items and tasks that are consistent with the vision of expertise presented in the CCSSM; b) augmenting them with additional apprentice-level items/tasks so as to *even out* the representation of mathematical practices, while simultaneously *concentrating the collection as a whole more deeply* on the major work of each grade;

and c) completing the assessment with a set of novice items/tasks that again collectively reflects the focus and coherence of the standards.

Strategic Uses of Technology: Wherever possible, computer-adaptive testing (CAT) is a desirable and efficient mechanism for testing and scoring. A range of selected response items (multiple-choice, drag-and-drop, and other categorization tasks) can be scored easily by computer, as can short constructed response items that require a straightforward answer, and many kinds of longer constructed response items. Much of a balanced assessment can be conducted using these tools. Technology also offers many powerful opportunities for working in mathematics, particularly the ability to rapidly and accurately perform large numbers of calculations and to both see and produce sophisticated visualizations. Appropriate use of such technology in assessment can improve balance in assessment by making higher-level thinking and understanding less expensive and more realistic (e.g. choosing the best statistical measures and representations for analyzing a data set with 1000 records, as opposed to selecting the median in a list of a dozen whole numbers).

"Technology enhanced" CAT tasks can also be designed to provide evidence for mathematical practices that could not be obtained from short/selected answer tasks, and can encourage classroom use of authentic mathematical computing tools (spreadsheets, interactive geometry, computer algebra, graphers) for classroom instruction.

At the same time, for much school-level mathematics, paper and pencil remains the natural medium for working mathematically, as it allows for diverse representations such as quick sketches of diagrams or graphs, and for mathematical expressions and tables to be rapidly created and freely mixed. Doing similar exploratory work on a computer would require the time-consuming use of multiple specialized tools, which were often designed for producing polished presentations or setting up large-scale computations rather than as a "scratchpad" for mathematical thinking. Sometimes only the end result of this work needs to be evaluated in the assessment – and it can be entered as an answer for computer scoring. At other times, the work itself is important to assess. For example, evaluating students' capacities to develop "multiple solution paths" and to "choose appropriate tools" requires an open-ended response format. Consequently, a useful blend of methods for working out problems and capturing students' mathematical ideas will be important to achieve.

SMARTER Balanced has accounted for this by planning for extended performance tasks to be administered beyond the CAT component of the tests. The current plan is to supplement the CAT test with a set of rich constructed response items (what are called here "expert tasks"), plus one classroom-based performance task (of up to 2 class periods). Examples of the range of item types needed to evaluate the standards are provided, with annotations regarding the standards they assess, in Part III, and the Appendix.

Part II – Overview of Claims and Evidence for CCSS Mathematics Assessment

Assessment Claims

The theory of action articulated in the Consortium's proposal to the U.S. Department of Education (http://www.k12.wa.us/SMARTER/pubdocs/SBAC_Narrative.pdf) illustrates the vision for an assessment system that will lead to inferences that ensure that all students are well-prepared for college and careers after high school. "Inference is reasoning from what one knows and what one observes, to explanations, conclusions, or predictions. One attempts to establish the weight and coverage of evidence in what is observed" (Mislevy, 1995, p 2). *Claims* are the broad statements of the assessment system's learning outcomes, each of which requires *evidence* that articulates the types of data/observations that will support interpretations of competence towards achievement of the claims. A first purpose of this document is to identify the critical and relevant claims that will "identify the set of knowledge and skills that is important to measure for the task at hand" (Pellegrino, Chudowsky, and Glaser, 2001), which in this case are the learning outcomes for the CCSS for mathematics.

After review from the field for this second round of the content specifications is received, analyzed, and integrated into a final version, the resulting claims for the mathematics assessment will be presented to the Smarter Balanced governing states for approval as Consortium policy. Governing state approval of the claims will ensure that all governing states have full endorsement of the major components of the summative assessments, and will establish those statements as the fundamental drivers for the design of the Consortium's summative assessments.

For this reason, within this document the claims stand out as being of particular significance. In fact, the other material presented here (in particular the Assessment Targets and the commentaries related to them) is meant to serve as general guidance and support for further development of the summative assessments. However, this additional material will not be subjected to endorsement by the governing states, and should not be viewed as Consortium policy. A more useful interpretation would be to view the Assessment Targets and commentaries as the "best thinking" of those who have contributed to this document, and should be considered as guidance for the further specifications of items and tasks and for the overall test design.

Four claims are proposed for the summative mathematics assessment. A detailed treatment of each claim follows in Part III, below. Each claim is summary statement about the knowledge and skill students will be expected to demonstrate on the assessment related to a particular aspect of the CCSS for mathematics. The level of the knowledge and skill necessary for a student to be proclaimed "Proficient" will be established through the development of Achievement Level Descriptors and during the setting of performance standards on the assessments.

Claims for Mathematics Summative Assessment

Claim #1	**Concepts & Procedures** "Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency."
Claim #2	**Problem Solving** "Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies."
Claim #3	**Communicating Reasoning** "Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others."
Claim #4	**Modeling and Data Analysis** "Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems."

Presentation of the Claims in Part III

Rationale for Claims: In Part III of this document, each claim is followed by a section describing what it is about this particular aspect of what students should know and be able to do that warrants a claim. The Rationale presents both the scope of the claim and its connection and alignment to the CCSS. In addition the claim is described in further detail than could be expected from the claim's single-sentence statement, and this description is provided in terms of what would be expected of a student who would demonstrate proficiency. In this way, the Rationale should be viewed as a starting point for the development of Achievement Level Descriptors.

Sufficient Evidence: Accompanying each claim in Part III is is a description of the sufficient relevant evidence from which to draw inferences or conclusions about student attainment of the claim. Relevant and sufficient evidence needs to be collected in order to support each claim. The assessment system will the opportunity to use a variety of assessment items and tasks applied in different contexts. It is important that the SMARTER Balanced pool of items and tasks for each claim be designed so the summative assessment can measure and be used to make interpretations about year-to-year student progress.

The sufficient evidence section for each claim includes a brief analysis of the assessment issues to be addressed to ensure accessibility to the assessment for all students, with particular attention to students with disabilities and English learners.

Assessment Targets: Finally, each claim is accompanied by a set of assessment targets that provide more detail about the range of content and Depth of Knowledge levels. The targets are intended to support the development of high-quality items and tasks that contribute evidence to the claims. We use the cluster level headings of the standards in the CCSS-M, in order to allow for the creation and use of assessment tasks that require proficiency in a broad range of content and practices. Use of more fine-grained descriptions would risk a tendency to atomize the content, which might lead to assessments that would not meet the intent of the standards. It is important to keep in mind the importance of developing items and tasks that reflect the richness of the mathematics in the MCCSS.

Proposed Reporting Categories

As used here, "Reporting Categories" define the levels of aggregation of score points on the assessment that will be reported *at the individual student level*. The paragraphs that follow identify the reporting categories that should be considered as a minimum goal of the assessment design. Nevertheless, constraints of logistics (e.g., cost and testing time) and/or psychometrics (e.g., dimensionality and stability of scales) may require a revision to what is proposed here. Although additional, more fine-grained reporting categories may be possible using aggregations (such as the classroom, school, and/or district levels), the feasibility of those score reporting categories will need to be evaluated once assessment blueprints have been established.[4]

First and foremost, because the summative assessment will be used for school, district, and state accountability consistent with current ESEA requirements, there needs to be a composite **"Total Mathematics" score** at the individual student level. Also, consistent with the SMARTER Balanced proposal and with requirements in the USED Notice Inviting Applications, the composite mathematics score will need to have scaling properties that allow for the valid determination of student growth over time. This score will be a weighted composite from the four claims, with Claim #1 (Concepts and Procedures) contributing roughly 40%, and with the three mathematical practices claims (#2 – Problem Solving; #3 – Communicating Reasoning; and #4 – Modeling and Data Analysis) contributing about 20% each.

Second, because of the central role of the claims in the design of the assessment, there should be **a reporting category for each claim**. Whether these are scaled scores or category classifications and whether or not growth should or can be evaluated on these scores cannot be determined until test blueprints have been established.

Finally, to ensure that results from the summative assessment can contribute to decisions that educators must make about patterns and trends in student learning, there need to be **reporting categories within Claim #1 (Concepts and Procedures)** relevant to the major domains at different grade levels. The CCSS provides a solid foundation for informing emphases on specific content at different grade levels. The major work of each grade, as defined in the Assessment Targets section for Claim #1 in Part III of this document, identifies the feasibility reporting at the domain level for each grade on the summative assessment. Additionally, since content domain level reporting categories will be reported only under Claim #1, content that is better assessed under other claims will likely not be reported as a domain sub-score, but will be utilized by students as they engage in mathematical practices. (Thus, for example,

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[4] Sireci, S.G. (2005). The Most Frequently Unasked Questions About Testing. In R. Phelps (Ed.), *Defending standardized testing* (pp. 111-121). Mahwah, NJ: Lawrence Erlbaum.

Geometry concepts will be assessed directly under # #1 at grade 8 where they are part of the grade's major emphases, while a significant portion of high school level Geometry content may be best assessed under Claims #2-4, as students use the content to engage in more complex mathematical practices.) The table below provides an overview of the summative mathematics assessment reporting categories for each grade. These reporting categories are grounded in evidence from international research, in terms of content coverage (areas of focus) and mathematical practices.[5] So, for example, a student in the 6th grade would receive a summative assessment report with seven scores: Total Mathematics; Concepts & Procedures; Number System; Ratio & Proportion; Expressions & Equations; Problem Solving; Communicating Reasoning; and Modeling and Data Analysis.

Proposed Reporting Categories for Summative Mathematics Assessment

<table>
<tr><td colspan="4">Total Mathematics Composite Score</td></tr>
<tr><td>Claim #1: Concepts and Procedures Score</td><td rowspan="8">Claim #2: Problem Solving Score</td><td rowspan="8">Claim #3: Communicating Reasoning Score</td><td rowspan="8">Claim #4: Modeling and Data Analysis Score</td></tr>
<tr><td><u>Grade 3 C&P Sub-scores</u>
Operations & Algebraic Thinking
Number/Ops – Fractions
Measurement & Data</td></tr>
<tr><td><u>Grade 4 C&P Sub-scores</u>
Operations & Algebraic Thinking
Number/Ops – Base 10
Number/Ops – Fractions
Measurement & Data</td></tr>
<tr><td><u>Grade 5 C&P Sub-scores</u>
Number/Ops – Base 10
Number/Ops – Fractions
Measurement & Data</td></tr>
<tr><td><u>Grade 6 C&P Sub-scores</u>
Number System
Ratio & Proportion
Expressions & Equations</td></tr>
<tr><td><u>Grade 7 C&P Sub-scores</u>
Number System
Ratio & Proportion
Expressions & Equations</td></tr>
<tr><td><u>Grade 8 C&P Sub-scores</u>
Expressions & Equations
Functions
Geometry</td></tr>
<tr><td><u>High School C&P Sub-scores</u>
Number & Quantity
Algebra
Functions</td></tr>
</table>

[5] Schmidt, W. H., Wang, C. H., & McKnight, C. C. 2005. Curriculum coherence: An examination of U.S. Mathematics and Science content standards from an international perspective, *Journal of Curriculum Studies, 37,* 525-559.

Part III – Detailed Rationale and Evidence for Each Claim

Mathematics Claim #1 **CONCEPTS AND PROCEDURES**
Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.

Rationale for Claim #1

This claim addresses procedural skills and the conceptual understanding on which developing skills depend. It is important to assess how aware students are of how concepts link together, and why mathematical procedures work in the way that they do. This relates to the structural nature of mathematics:

> Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression x2 + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. (Practice 7, CCSSM)
>
> They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. (Practice 7, CCSSM)
>
> Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y – 2)/(x – 1) = 3. Noticing the regularity in the way terms cancel when expanding (x – 1)(x + 1), $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. (Practice 8, CCSM)

Assessments should include items/tasks that test the precision with which students are able to carry out procedures, describe concepts and communicate results.

> Mathematically proficient students … state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. (Practice 6, CCSSM)

Items/tasks should also assess how well students are able to use appropriate tools strategically.

> Students are able to use technological tools to explore and deepen their understanding of concepts. (Practice 5; CCSSM)

Many individual content standards in CCSSM set an expectation that students can *explain why* given procedures work.

> One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as (a + b)(x + y) and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding (a + b + c)(x + y). Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness. (CCSSM, p.4).

Finally, throughout the K-6 standards in CCSSM there are also individual content standards that set expectations for fluency in computation (e.g., fluent multiplication and division within the times tables in Grade 3). Such standards are culminations of progressions of learning, often spanning several grades, that involve conceptual understanding, thoughtful practice, and extra support where necessary. Technology may offer the promise of assessing fluency more thoughtfully than has been done in the past. This, too, is part of 'measuring the full range of the standards.'

Following our discussion of the types of evidence appropriate for contributing to assessment of Claim #1, we describe specific grade-level content emphases.

What sufficient evidence looks like for Claim #1

Evidence on each student's progress along the progressions of mathematical content is the focus of attention in assessing this claim.

Essential properties of items and tasks that assess this claim: Items and tasks that could provide evidence for this claim include brief items – selected response and short constructed response items – that focus on a particular procedural skill or concept. Brief items could also include items that require students to translate between or among representations of concepts (words, diagrams, symbols) and items that require students to identify an underlying structure. Brief constructed response items can include items that provide scaffolded support for the student; it is probably possible for a Computer Adaptive environment to adjust the level of scaffolding that is provided depending on the student's performance level.

Selected response items, including computer-enhanced items, can probe conceptual understanding, particularly when the distractors are chosen to embody common misconceptions. In designing such items, it is essential to try to make sure that students do not obtain correct answers because of "test taking skills" rather than understanding of the mathematical content. Computer administration of the assessment affords the possibility of assessing student fluency with mathematical operations by means of monitoring the response time.

Short Constructed response items can assess mathematical thinking directly; short items of this kind can provide direct evidence on students' mastery of standard procedures. Among items/tasks that require students to produce a response, short constructed response items are the most likely to be able to be machine scored.

Highly scaffolded tasks, where the student is guided through a series of short steps set in a common problem context, offer another approach to the design of short constructed response items.

Extended Response items, requiring a more solid demonstration of conceptual understanding and procedural skills that students may be expected to have learned and practiced, may also provide evidence for this claim. These can include the following task types**:**

- **Application tasks** using exercises to assess relatively standard applications of mathematical principals. Here, students can be expected to use important concepts and skills to tackle problem situations that should be in the learned part of the curriculum.
- **Translation tasks**, where students are asked to represent concepts in different ways and translate between representations (words, numbers, tables, graphs, symbolic algebra).
- **Explanation tasks,** where students are asked to explain why a given standard procedure works. This may involve the straightforward adaptation of a standard procedure.

Accessibility & Claim #1: This claim clarifies the importance of conceptual understanding and procedural knowledge underlying the important core content in CCSSM. The standards refer to the ability to carry out procedures, describe concepts, communicate results, use appropriate tools strategically, and explain why specific procedures make sense. Neither the claim itself nor the CCSSM explicitly address the challenges that some students with disabilities face in the area of mathematical calculations. Because of the importance of building skills in computation in early schooling, the explication of the content may be different in early school grades compared to later school grades. Providing assistive technologies such as an abacus or calculator may not be considered appropriate up through about grade 4. At some point during intermediate grades, the use of these tools is considered an appropriate avenue of access to allow students to demonstrate that they are able to "calculate accurately and efficiently.

It is also important to address access to mathematics via decoding text and written expression. The uses of alternative means of access and expression are ones used by successful individuals (Reitz, 2011) to demonstrate high levels of success, and thus are an appropriate avenue of access to the content for students with disabilities in the areas of reading decoding and fluency as well as for those with blindness or visual impairments. Likewise, allowing students alternative ways to express their understanding of mathematics content is important. Students who are unable to explain mathematical processes via writing or computer entry might instead provide their explanation via speech to text technology (or a scribe) or via manipulation of physical objects.

A major aspect of all the claims, including Claim #1, is communication, especially students' ability to explain *why or how* given procedures or approaches work. To allow access to English learners who are at a lower proficiency in writing and speaking, it will be important that explanations allow the use of diagrams, drawings, equations, and mathematical models, as well as words. It will also be useful to provide opportunities for ELL students to communicate their understanding through performance tasks or other approaches where multiple domain input can be provided. Furthermore, when a major performance difference exists between tasks such as expanding and explaining, it will be important to allow students to express their views through the use of native language, where that is appropriate.

Assessment Targets for Claim #1

Cluster headings as assessment targets: Cluster headings often serve to communicate the larger intent of a group of standards. For example, a cluster heading in Grade 4 reads: "Generalize understanding of place value for multi-digit numbers." Individual standards in this cluster pinpoint some signs of success in the endeavor, but the important endeavor itself is stated directly in the cluster heading. In addition, the word "generalize" signals that there is a multi-grade progression in grades K-3 leading up to this group of standards. In ways such as these, the cluster headings often best communicate the focus and coherence of the standards. Therefore, this specification document *uses the cluster headings as the targets of assessment* for generating evidence for Claim #1. For each cluster, specifications are provided that give item developers important guidance about task design for the cluster. A series of example items will also be provided that illustrate the content scope and range of difficulty appropriate to assessing the cluster. Claim #1 assessment targets are shown below for Grades 3, 5 and 8. Content emphases for the remaining grades are shown in Appendix A. Assessment targets for these other grades will be developed after allowing the field to provide feedback on the current draft.

Content emphases in the standards: Not all content is emphasized equally in the Standards for Mathematical Content, and this is in keeping with the design principles of focus and coherence in the standards as a whole.

- The standards communicate emphases in many ways, including by the use of domain names that vary across the grades, and that are sometimes much more fine-grained than the top-level organizers in previous state standards (e.g., Ratios and Proportional Relationships). These and other features of the standards and their progressions point to the major work of each grade.[6]

- Meanwhile, standards for topics that are not major emphases in themselves are generally written in such a way as to support and strengthen the areas of major emphasis. This promotes valuable connections that add coherence to the grade.

- Finally, still other topics that may not connect tightly or explicitly to the major work of the grade would fairly be called additional.

In the tables that follow and in Appendix A, these designations—major, additional, and supporting—are provided at the cluster level.

Working at the cluster level helps to avoid obscuring the big ideas and getting lost in the details of specific standards (which are individually important, but impossible to measure in their entirety within the bounds of reasonable testing time). Clusters work as an appropriate grain size for following the contours of important progressions in the standards, for example the integration of place value understanding and the meanings and properties of operations that must happen as students develop computation strategies and algorithms for multi-digit numbers during grades K-6; or the appropriate development of functional thinking in middle school leading to the emergence of functions as a content domain in Grade 8.

To say that the standards do not emphasize everything equally is not to say that anything in the standards can be neglected; to do so would leave gaps in student preparation for later mathematics. All content is therefore eligible for assessment. However, evidence for Claim #1 will strongly focus on the major clusters and take into account ways in which the standards tie supporting clusters to the major work of each grade. The content footprint of any given test will sample in much greater proportion from clusters representing the major work of each grade.

For Claim #1 Assessment Targets are provided for three representative grade levels – Grades 3, 5, and 8. Targets for Grades 4, 6, 7, and High School will be completed after feedback on this draft of the Content Specifications is received and analyzed.

[6] Further emphases can be seen in the *Progressions* documents drafted by members of the Common Core State Standards Working Group, and published through the Institute for Mathematics and Education of the University of Arizona: http://ime.math.arizona.edu/progressions/

GRADE 3 Summative Assessment Targets
Providing Evidence Supporting Claim #1

Claim #1: Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content for this claim may be drawn from any of the Grade 3 clusters represented below, with a much greater proportion drawn from clusters designated “m” (major) and the remainder drawn from clusters designated “a” (additional) and “s” (supporting) – with supporting items usually connecting to the major work of the grade. Sampling of Claim #1 assessment targets will be determined by balancing the content assessed with items and tasks for Claims #2, #3, and #4.[7]

Operations and Algebraic Thinking

Target A [m]: Represent and solve problems involving multiplication and division.[8] (DOK 1, 2)
Tasks for this target require students to use multiplication and division within 100 to solve straightforward, one-step contextual word problems in situations involving equal groups, arrays, and measurement quantities such as length, liquid volume, and masses/weights of objects. The majority of these problems should be of the equal groups and arrays situation types, with the more difficult measurement quantity situations in the minority. All of these tasks will code straightforwardly to standard 3.OA.3. Few of these tasks coding to this standard will make the method of solution a separate target of assessment. Other tasks associated with this target will probe student understanding of the meanings of multiplication and division (3.OA.1,2).[9]

Non-contextual tasks that explicitly ask the student to determine the unknown number in a multiplication or division equation relating three whole numbers (3.OA.4) will support the development of items that provide a range of difficulty necessary for populating an adaptive item bank (see section *Understanding Assessment Targets in an Adaptive Framework*, below, for further explication.).

Target B [m]: Understand properties of multiplication and the relationship between multiplication and division. (DOK 1)
Whereas Target A focuses more on the practical uses of multiplication and division, Target B focuses more on the mathematical properties of these operations, including the mathematical relationship between multiplication and division. Tasks associated with this target are not intended to be vocabulary exercises along the lines of “which of these illustrates the distributive property?” As indicated by the CCSSM,[10] students need not know the formal names for the properties of operations. Instead, tasks are to probe whether students are able to *use* the properties to multiply and divide.
Note, tasks that code directly to Target B will be limited to the 10x10 times table. (But see Target A under 3.NBT below.)

Target C [m]: Multiply and divide within 100. (DOK 1)
The primary purpose of tasks associated with this target is to assess fluency and/or memory within the

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[7] For example, if under Claim #2, a problem solving task in a given year centers on a particular topic area, then it is unlikely that this topic area will also be assessed under Claim #1 in a selected response item.
[8] See CCSSM, Table 2, p. 89 for additional information.
[9] Note the examples given in italics in CCSSM for these two standards. [CCSSM p. 23]
[10] See CCSSM, footnote on page 23.

10x10 times table. We note that the standard connotation of the word "fluency" with regard to standards such as 3.OA.7 means "quickly and accurately."[11] An expansion of this concept would be useful, to include both the ability to use certain facts and procedures with enough facility that using them does not slow down or derail the problem solver as he or she works on more complex problems, *and* the notion of conceptual fluency - being able to use the relevant ideas or procedures in a wide range of contexts. In an adaptive framework, straight multiplication and division problems that assess students' ability to multiply and divide within 100 may serve as the assessment floor for the Operations and Algebraic Thinking domain (See section *Understanding Assessment Targets in an Adaptive Framework)*.

Target D [m]: Solve problems involving the four operations, and identify and explain patterns in arithmetic. (DOK 2)
These tasks will primarily consist of contextual word problems requiring more than a single operation or step. Most of these will be straightforward two-step contextual word problems coding straightforwardly to 3.OA.8. These problems serve an important purpose in showing that students have solidified addition and subtraction problem solving from previous grades and integrated it correctly alongside their new understandings of multiplication and division.

Multiplication and division steps should be limited to the 10x10 times table, but addition and subtraction steps should often involve numbers larger than 100 (cf. 3.NBT.2).

In some tasks associated with this target, the representation of the problem with equations and/or the judgment of the reasonableness of an answer should be the explicit target for the task (cf. 3.OA.8).

Number and Operations—Base Ten

Target A [a]: Use place value understanding and properties of arithmetic to perform multi-digit arithmetic. (DOK 1)
Tasks associated with this target will be non-contextual computation problems that assess fluency in addition and subtraction within 1000.[12] Some of these tasks should provide information about the strategies and/or algorithms students are using, in order to ensure that they are general (based on place value and properties of operations).

Other tasks will assess either rounding (with an emphasis on conceptual understanding, if possible) or the more important multi-digit computations specified in 3.NBT.3. Because the answers to such multiplications are easily found by mnemonic tricks, these items should be of a conceptual nature to assess reasoning with place value and properties of operations.

[11] In other words, this standard does not refer to *procedural fluency* as that term is used in Claim #1 generally. (See *Adding It Up: Helping Children Learn Mathematics.* NRC, 2001, p. 121.)
[12] The word "fluently" in standard 3.NBT.2 means "quickly and accurately" rather than referring to procedural fluency as that term is used in Claim #1 generally. (See *Adding It Up: Helping Children Learn Mathematics.* NRC, 2001, p. 121.)

Number and Operations—Fractions

Target A [m]: Develop understanding of fractions as numbers. (DOK 1, 2)
Some of these tasks should assess conceptual understanding of unit fractions and other fractions as detailed in 3.NF.1 and 3.NF.2.[13] Other tasks for this cluster should involve equivalence of fractions as detailed in 3.NF.3. Tasks should attempt to cover the range of expectations in the standard, such as understanding, recognizing, generating, and expressing, although explanations and justifications may also be assessed under Claim #3.

The cluster heading refers to understanding fractions as numbers. To assess whether students have met this goal, tasks for this target should include fractions greater than 1 as well as fractions less than or equal to 1; and tasks should not handle fractions differently based on whether they are greater than, less than, or equal to 1. Fractions equal to whole numbers (such as 3/1) should also commonly appear in these tasks. Two equal fractions may be referred to as equal, without need for the term "equivalent" (e.g., "which fraction equals 3?"), and fractions may be referred to simply as numbers (e.g., "which number is greatest?" with fractions among the answer choices).

Measurement and Data

Target A [m]: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. (DOK 1, 2)
Tasks for this target generally require students to solve straightforward one-step contextual word problems using the four operations in a situation involving time intervals in minutes, liquid volume in liters, and mass/weight in grams and kilograms. Situations involving intervals of time are limited to addition and subtraction.[14] Some foundational tasks that assess telling and writing time to the nearest minute may be appropriate for building a range of difficulty in the adaptive item bank. The emphasis for this target is not on cultural aspects of time such as clocks but rather on time as a measurement quantity that can be operated on arithmetically like other more tangible measurement quantities.

Target B [s]: Represent and interpret data. (DOK 2, 3)
Tasks associated with this target should involve using information presented in scaled bar graphs to solve one- and two-step "how many more" and "how many less" problems.[15] Technology might be used to enable students to draw a scaled picture graph and a scaled bar graph to represent a data set with up to four categories. Other tasks can involve the cycle indicated in 3.MD.4 (measure to generate data, and show the data by making a line plot); such tasks should indeed involve fractional measurement values.

[13] Note that area models, strip diagram models, and number line models of a/b are all essentially special cases of the core fraction concept as defined in 3.NF.1: namely, a parts when a whole is partitioned into b equal parts. In the case of a number line, the "whole" in question is the interval from 0 to 1.
[14] Tasks for this target will not involve fractional quantities. Tasks will not require students to distinguish between mass and weight. Tasks will exclude compound units such as cm3 and exclude finding the geometric volume of a container. (See CCSSM page 25 footnote 6.) Tasks will not include "times as much" problems (cf. 4.OA.1,2 and CCSSM Glossary Table 2, p. 89).
[15] The "uptick" in this progression from Grade 2 is that in Grade 2, bar graphs are not scaled. The introduction of scaled graphs in Grade 3 connects with the introduction of multiplication in Grade 3.

Target C [m]: Geometric measurement: understand concepts of area and relate area to multiplication and to addition. (DOK 1, 2)
Some tasks associated with this target should assess conceptual understanding of area as a measurable attribute of plane figures. All figures in such problems should be rectilinear and coverable without gaps or overlaps by a whole number of unit squares without having to dissect the unit squares (e.g. partition them into two triangles). Tasks in this group will generally involve finding areas by direct counting of unit squares, not by using multiplication or formulas, or otherwise reasoning about areas on this basis.

Other tasks should center on relating area to multiplication and addition. Most of these should involve the use of area models to represent whole-number products and the distributive property. For example, "Draw a picture to show why Amber can add 5x5 and 2x5 to find 7x5." Problems can involve finding areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts.

Some of the expectations in this cluster (such as using tiling to show that area of a rectangle with whole-number side lengths is the same as would be found by multiplying the side lengths) may be more suitable for Claims #3 and #4 or for in-class assessments.

Target D [a]: Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures. (DOK 1)
Tasks associated with standard (3.MD.8) will assess students' ability to solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Geometry

Target A [s]: Reason with shapes and their attributes. (DOK 1, 2)
These tasks should support Grade 3 fraction and area work. Technology-enhanced tasks could involve partitioning a shape into parts with equal areas; more traditional tasks could involve expressing the area of each part as a unit fraction of the whole. For these tasks, shapes may be partitioned into non-rectangular parts; for example, students will use intuitive ideas about area to reason that a square with both diagonals drawn has been partitioned into four equal parts.[16]

Other tasks for this target will connect less directly to other material in the grade, continuing instead the standards' progression of increasingly sophisticated spatial and logical reasoning about shapes and their attributes (cf. 2.G.1). Most of these tasks will assess understanding of the hierarchy of quadrilaterals as detailed in 3.G.1. A few tasks may involve categories of shapes not explicitly mentioned in the standard, so as to assess understanding of property-based categorization per se at this level. For example, a regular octagon and a rectangle might be shown and the student asked to select a category to which both figures belong—e.g., figures that can be partitioned into triangles—and then to produce a figure not belonging to that category (e.g., a circle).

[16] Cf. standard 2.G.3. See also the figure at top of page 3 in the draft *Progression* on fractions, http://commoncoretools.files.wordpress.com/2011/08/ccss_progression_nf_35_2011_08_12.pdf.

Grade 5 SUMMATIVE ASSESSMENT TARGETS
Providing Evidence Supporting Claim #1

Claim #1: Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content for this claim may be drawn from any of the Grade 3 clusters represented below, with a much greater proportion drawn from clusters designated "m" (major) and the remainder drawn from clusters designated "a" (additional) and "s" (supporting) – with supporting items usually connecting to the major work of the grade. Sampling of Claim #1 assessment targets will be determined by balancing the content assessed with items and tasks for Claims #2, #3, and #4.[1]

Operations and Algebraic Thinking

Target A [a]: Write and interpret numerical expressions. (DOK 1)
Tasks for this target will require students to write expressions to express a calculation and evaluate and interpret expressions. Some of these tasks should incorporate the work of using the associative and distributive properties in writing and evaluating expressions, but expressions will not contain nested grouping symbols.

Target B [a]: Analyze patterns and relationships. (DOK 2)
Tasks for this target will ask students to compare two related numerical patterns and explain the relationships within sequences of ordered pairs. Tasks for this target may incorporate the work of 5.G Target A.

Number and Operations—Base Ten

Target A [m]: Understand the place value system. (DOK 1, 2)
Tasks for this target ask students to explain patterns in the number of zeros for powers of 10, including simple calculations with base 10 and whole number exponents as well as tasks that demonstrate a generalization of the pattern for larger whole number exponents (e.g., How many zeros would there be in the answer for 10^{42}?).

Other tasks for this target ask students to write, compare, and round decimals to thousandths. Some decimals should be written in expanded form. Comparing and rounding may be combined in some items to highlight essential understandings of connections (e.g., What happens if you compare 3.67 and 3.72 after rounding to the nearest tenth?).

Target B [m]: Perform operations with multi-digit whole numbers and with decimals to hundredths. (DOK 1, 2)
Some tasks associated with this target will be non-contextual computation problems that assess fluency in multiplication of multi-digit whole numbers.[1]

Other tasks will ask students to find quotients of whole numbers with up to four-digit dividends and two-digit divisors and use the four operations on decimals to hundredths. These tasks may be presented in the context of measurement conversion (5.MD Target A). Other tasks should highlight students' understanding of the relationships between operations and use of place value strategies, which may be done as part of tasks developed for Claim #3.

Number and Operations—Fractions

Target A [m]: Use equivalent fractions as a strategy to add and subtract fractions. (DOK 1)
Tasks associated with this target ask students to add and subtract fractions with unlike denominators, including mixed numbers. Contextual word problems that ask students to apply these operations should be included (often paired with one or more targets from Claim #2). Other tasks should focus on the reasonableness of answers to addition and subtraction problems involving fractions, often by presenting "flawed reasoning" (paired with one or more targets from Claim #3).

Target B [m]: Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (DOK 1, 2)
Tasks for this target will ask students to multiply and divide fractions, including division of whole numbers where the answer is expressed by a fraction or mixed number. Division tasks should be limited to those that focus on dividing a unit fraction by a whole number or whole number by a unit fraction. Extended tasks posed as real world problems related to this target will be assessed with targets from Claims #2 and #4.

Other tasks will ask students to find the area of a rectangle with fractional side lengths or use technology enhanced items to build visual models of multiplication of fractions, where the student is able to partition and shade circles or rectangles as part of an explanation.
Students' ability to interpret multiplication as scaling will be assessed with the targets for Claim #3.

Measurement and Data

Target A [s]: Convert like measurement units within a given measurement system. (DOK 1)
Tasks for this target ask students to convert measurements and should be used to provide context for the assessment of 5.NBT Target B. Some tasks will involve contextual problems and will contribute evidence for Claim #2 or Claim #4.

Target B [s]: Represent and interpret data. (DOK 1, 2)
Tasks for this target ask students to make and interpret line plots with fractional units and should be used to provide context for the assessment of 5.NF Target A and 5.NF Target B. Some tasks will involve contextual problems and will contribute evidence for Claim #2 or Claim #4.

Target C [m]: Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition. (DOK 1, 2)
Tasks for this target will ask students to find the volume of right rectangular prisms with whole number edge lengths using unit cubes and formulas. Some tasks should ask students to consider the effect of changing the size of the unit cube (e.g., doubling the edge length of a unit cube) using values that do not cause gaps or overlaps when packed into the solid. Other tasks will ask students to find the volume of two non-overlapping right rectangular prisms, often together with targets from Claim #2 or #4.

Geometry

Target A [a]: Graph points on the coordinate plane to solve real-world and mathematical problems. (DOK 1)
Tasks for this target ask students to plot coordinate pairs in the first quadrant. Some of these tasks will be created by pairing this target with 5.OA Target B, which would raise the DOK level.

Target B [a]: Classify two-dimensional figures into categories based on their properties. (DOK 2)
Tasks for this target ask students to classify two-dimensional figures based on a hierarchy. Technology enhanced items may be used to construct a hierarchy or tasks may ask the student to select all classifications that apply to a figure based on given information.

Grade 8 SUMMATIVE ASSESSMENT TARGETS Providing Evidence Supporting Claim #1
Claim #1: Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Content for this may be drawn from any of the Grade 3 clusters represented below, with a much greater proportion drawn from clusters designated "m" (major) and the remainder drawn from clusters designated "a" (additional) and "s" (supporting) – with supporting items usually connecting to the major work of the grade. Sampling of Claim #1 assessment targets will be determined by balancing the content assessed with items and tasks for Claims #2, #3, and #4.[17]

The Number System

Target A [s]: Know that there are numbers that are not rational, and approximate them by rational numbers. (DOK 1)
Tasks for this target will require students to convert between rational numbers and decimal expansions of rational numbers.

Other tasks will ask students to approximate irrational numbers on a number line or as rational numbers with a certain degree of precision. This target may be combined with 5.EE Target A (e.g., by asking students to express the solution to a cube root equation as a point on the number line).

Expressions and Equations

Target A [m]: Work with radicals and integer exponents. (DOK 1)
Tasks for this target will require students to select or produce equivalent numerical expressions based on properties of integer exponents.

Other tasks will ask students to solve simple square root and cube root equations, often expressing their answers approximately using one of the approximations from 5.NS Target A.

Other tasks will ask students to represent very large and very small numbers as powers of 10, including scientific notation, and perform operations on numbers written as powers of 10.

Target B [m] Understand the connections between proportional relationships, lines, and linear equations. (DOK 2)
Tasks for this target will ask students to graph one or more proportional relationships and connect the unit rate(s) to the context of the problem.

Other tasks will ask students to apply understanding of the relationship between similar triangles and slope.[18]

Target C [m]: Analyze and solve linear equations and pairs of simultaneous linear equations. (DOK 2)
Tasks for this target will ask students to solve systems of two linear equations in two variables algebraically and estimate solutions graphically. Some problems will ask students to recognize simple cases of two

[17] For example, if under claim #2, a problem solving task in a given year centers on a particular topic area, then it is unlikely that this topic area will also be assessed under claim #1 in a selected response item.
[18] For example, a task might say that starting from a point on a line, a move ¾ to the right and one unit up puts you back on the line. If you start at a different point on the line and move to the right 8 units, how many units up do you have to move to be back on the line?

equations that represent the same line or that have no solution. This target may be combined with 8.F Target B to create problems where students determine a point of intersection given an initial value and rate of change, including cases where no solution exists.

Real world and mathematical problems that lead to two linear equations in two variables will be assessed in connection with targets from Claims 2 and 4.

Functions

Target A [m]: Define, evaluate, and compare functions. (DOK 1, 2)
Tasks associated with this target ask students to relate different functional forms (algebraically, graphically, numerically in tables, or by verbal descriptions). Some tasks for this target will ask students to produce or identify input and output pairs for a given function. Other tasks will ask students to compare properties of functions (e.g., rate of change or initial value).

Other tasks should ask students to classify functions as linear or non-linear when expressed in any of the functional forms listed above. Some of these may be connected to 8.SP Target A.

Target B [s]: Use functions to model relationships between quantities. (DOK 1, 2)
Technology enhanced items will ask students to identify parts of a graph that fit a particular qualitative description (e.g., increasing or decreasing) or sketch a graph based on a qualitative description.

Other tasks for this target will ask students to determine the rate of change and initial value of a function from given information. Some tasks will ask students to give the equation of a function that results from given information.

Geometry

Target A [m]: Understand congruence and similarity using physical models, transparencies, or geometry software. (DOK 2)
Technology enhanced items will be used to allow students to "draw" lines, line segments, angles, and parallel lines after undergoing rotations, reflections, and translations. Similar technology enhanced items will ask students to produce a new figure or part of a figure after undergoing dilations, translations, rotations, and/or reflections.

Other tasks will present students with two figures and ask students to describe a series of rotations, reflections, translations, and/or dilations to show that the figures are similar, congruent, or neither. Many of these tasks will contribute evidence for Claim #3, asking students to justify their reasoning or critique given reasoning within the task.

Target B [m]: Understand and apply the Pythagorean theorem. (DOK 2)
Tasks associated with this target will ask students to use the Pythagorean Theorem to solve real-world and mathematical problems in two and three dimensions, including problems that ask students to find the distance between two points in a coordinate system.

Some applications of the Pythagorean Theorem will be assessed at deeper levels in Claims #2 and #4. Understanding of the derivation of the Pythagorean Theorem would contribute evidence to Claim #3.

Target C [a]: Solve real-world and mathematical problems involving volume of cylinders, cones and spheres. (DOK 2)
Tasks for this target will ask students to apply the formulas for volume of cylinders, cones and spheres to solve problems. Many of these tasks will contribute evidence to Claims #2 and #4.

Statistics and Probability

Target A [s]: Investigate patterns of association in bivariate data. (DOK 1, 2)
Tasks for this target will often be paired with 8.F Target B and ask students to determine the rate of change and initial value of a line suggested by examining bivariate data. Interpretations related to clustering, outliers, positive or negative association, linear and nonlinear association will primarily be presented in context by pairing this target with those from Claims #2 and #4.

Understanding Assessment Targets in an Adaptive Framework: In building an adaptive test, it is essential to understand how content gets "adapted." In a computer adaptive summative assessment, it doesn't make much sense to repeatedly offer formulaic multiplication and division items to a highly fluent Grade 3 student, making the Grade 3 Target OA.C [m] less relevant for this student than it may be for another. The higher-achieving student could be challenged further, while a student who is struggling could be given less complex items to ascertain how much each understands within the domain. The table below illustrates several items for the Grade 3 Operations and Algebraic Thinking domain that would likely span the difficulty spectrum for this grade. The items generally get more difficult with each row (an important feature of adaptive test item banks). (Pilot data will be used to determine more precisely the levels of difficulty associated with each kind of task.)

Sample for Grade 3, Claim #1 – Operations and Algebraic Thinking

Adapting Items within a Claim & Domain	Claim #1 – Operations and Algebraic Thinking
8 x 5 = □	Target C [m]: Multiply and divide within 100.
8 x □ = 40	Target A [m]: Represent and solve problems involving multiplication and division.
9 x 4 = □ x 9	Target B [m]: Understand properties of multiplication and the relationship between multiplication and division.
4 x 2 x □ = 40	Target B [m]: Understand properties of multiplication and the relationship between multiplication and division.
4 x 2 x □ = 5 x 2 x 2 x 2	Target B [m]: Understand properties of multiplication and the relationship between multiplication and division.
9 x 4 = 4 x □ x □ (May appear as a drag and drop TE item	Target B [m]: Understand properties of multiplication and the relationship between multiplication and division.

where "1" is not one of the choices for dragging.)	
8 x □ = 4 x □ Give two different pairs of numbers that could fill the boxes to make a true equation (selected response, drag and drop, or fill-in would work).	Target B [m]: Understand properties of multiplication and the relationship between multiplication and division.

Some of the more difficult items in the table incorporate several elements of this potential Grade 3 progression (fluency with multiplication → understanding the "unknown whole number" in a multiplication problem → applying properties of operations). Thus, a student who is consistently successful with items like the one in the final rows would not necessarily be assessed on items in previous rows within an adaptive test. In this way adaptive testing has the benefit of reduced test length while providing coverage of a broad scope of knowledge and skills. Adapting to greater and lesser difficulty levels than those illustrated in the table may require the use of items from other grades.

The relative impact of a student's ability or inability to "multiply and divide within 100" (Target C) would likely affect his/her performance on other clusters in the domain of Operations and Algebraic Thinking, thus serving as a baseline for much of the other content in this domain.

The sample items in the table illustrate another point – that the cluster level of the CCSS provides a suitable grain size for the development of a well-supplied item bank for computer adaptive testing. Item quality should not be compromised, particularly in an adaptive framework, by unnecessarily writing items at too fine a grain size. Since efficiency and appropriate item selection are optimized by minimizing constraints on the adaptive test (Thompson & Weiss, 2011), it is critical to ensure that items provide an appropriate range of difficulty within each domain for Claim #1.

Mathematics Claim #2
PROBLEM SOLVING

Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.

Assessment items and tasks focused on this claim include well-posed problems in pure mathematics and problems set in context. *Problems* are presented as items and tasks that are well posed (that is, problem formulation is not necessary) and for which a solution path is not immediately obvious.[19] These problems require students to construct their own solution pathway, rather than to follow a provided one. Such problems will therefore be unstructured and students will need to select appropriate conceptual and physical tools to use.

Rationale for Claim #2

At the heart of doing mathematics is making sense of problems and persevering in solving them[20]. This claim addresses the core of mathematical expertise – the set of competences that students can use when they are confronted with challenging tasks.

"Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary." (Practice 1, CCSSM)

Problem solving, which of course builds on a foundation of knowledge and procedural proficiency, sits at the core of *doing* mathematics. Proficiency at problem solving requires students to choose to use concepts and procedures from across the content domains and check their work using alternative methods. As problem solving skills develop, student understanding of and access to mathematical concepts becomes more deeply established.

For example, "older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need.

[19] Schoenfeld, A. H. (1985). *Mathematical problem solving.* Orlando, FL: Academic Press.
[20] See, e.g., Halmos, P. (1980). The heart of mathematics. *American Mathematical Monthly*, 87, 519-524

Mathematically proficient students can approach and solve a problem by drawing upon different mathematical characteristics, such as: correspondences among equations, verbal descriptions of mathematical properties, tables graphs and diagrams of important features and relationships, graphical representations of data, and regularity or irregularity of trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.” (Practice 1, CCSSM)

Development of the capacity to solve problems also corresponds to the development of important meta-cognitive skills such as oversight of a problem-solving process while attending to the details. Mathematically proficient students continually evaluate the reasonableness of their intermediate results, and can step back for an overview and shift perspective. (Practice 7, Practice 8, CCSM)

Problem solving also requires students to identify and select the tools that are necessary to apply to the problem. The development of this capacity – to frame a problem in terms of the steps that need to be completed and to review the appropriateness of various tools that are available – are critical to further learning in mathematics, and generalize to real-life situations. This includes both mathematical tools and physical ones:

“Tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge.” (Practice 5, CCSSM)

What sufficient evidence looks like for Claim #2

Although items and tasks designed to provide evidence for this claim must primarily assess the student’s ability to identify the problem and to arrive at an acceptable solution. Nevertheless, mathematical problems require students to apply mathematical concepts and procedures. Thus, though the primary purpose of items/tasks associated with this claim is assess problem solving skill, these items could possibly also contribute to evidence that is gathered for Claim #1.

Properties of items/tasks that assess this claim: The rationale for this claim makes it clear that evidence for it needs to include student demonstration of actual application of problem solving. The assessment of many relatively discrete and/or single-step problems can be accomplished using short constructed response items, or even computer-enhanced or selected response items.

Additionally, more extensive constructed response items can effectively assess multi-stage problem solving and can also indicate unique and elegant strategies used by some students to solve a given problem, and can illuminate flaws in student's approach to solving a problem. These tasks could:

- Present non-routine[21] problems where a substantial part of the challenge is in deciding what to do, and which mathematical tools to use; and
- Involve chains of autonomous[22] reasoning, in which some tasks may take a successful student 5 to 10 minutes, depending on the age of student and complexity of the task.

A distinctive feature both selected response items and extended response tasks for Claim #2 is that they are "well-posed". That is, whether the tasks deal with pure or applied contexts, the problem itself is completely formulated; the challenge is in identifying or using an appropriate solution path. Consider the following example, where the students may select a numerical, algebraic or graphical approach.

It is recognized that such tasks will be new to many students. *For some of these tasks,* therefore, it might be worthwhile to explore the development of scaffolding supports within the assessment to facilitate entry and assess student progress towards expertise. The degree of scaffolding for individual students might be able to be determined as part of the adaptability of the computer-administered test. Even for such "scaffolded" tasks," part of the task will involve a chain of autonomous reasoning that could still take a student 5 to 10 minutes to complete. Some tasks might present significant cognitive demand on most students. For this reason consideration should be given to framing more complex problem solving tasks with mathematical concepts and procedures that have been mastered in an earlier grade.

Scoring rubrics for extended response items and tasks should be consistent with the expectations of this claim, giving substantial credit to the choice of appropriate methods of tackling the problem, to reliable skills in carrying it through, and to explanations of what has been found.

Accessibility and Claim #2: This claim about mathematical problem solving focuses on the student's ability to make sense of problems, construct pathways to solving them, persevering in solving them, and the selection and use of appropriate tools. This claim includes student use of appropriate tools for solving mathematical problems, which for some students may extend to tools that provide full access to the item or task and to the development of reasonable solutions. For example, students who are blind and use Braille or assistive technology such as text readers to access written materials, may demonstrate their modeling of physical objects with geometric shapes using alternate formats. Students who have physical disabilities that preclude movement of arms and hands should not be precluded from demonstrating with assistive technology their use of tools for constructing shapes. As with Claim #1, access via text to speech and expression via scribe, computer, or speech to text technology will be important avenues for enabling many students with disabilities to show what they know and can do in relation to framing and solving complex mathematical problems.

[21] As noted earlier, by "non-routine" we mean that the student will not have been taught a closely similar problem, so will not expect to *remember* a solution path but to have to *adapt* or *extend* their earlier knowledge to find one.
[22] By "autonomous" we mean that the student responds to a single prompt, without further guidance within the task.

With respect to English learners, the expectation for verbal explanations of problems will be more achievable if formative materials and interim assessments provide illustrative examples of the communication required for this claim, so that ELL students have a better understanding of what they are required to do. In addition, formative tools can help teachers teach ELL students ways to communicate their ideas through simple language structures in different language modalities such as speaking and writing. Finally, attention to English proficiency in shaping the delivery of items (e.g. native language or linguistically modified, where appropriate) and the expectations for scoring will be important.

Assessment Targets for Claim #2

Claim #2 is aligned to the mathematical practices from the MCCSS, which are consistent across grade levels. For this reason, the Assessment Targets are not divided into a grade-by-grade description. Rather, a general set of targets is provided, which can be used as guidance for the development of item and test specifications for each grade.

SUMMATIVE ASSESSMENT TARGETS **Providing Evidence Supporting Claim #2**
Claim #2: Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.
To preserve the focus and coherence of the standards as a whole, tasks must draw clearly on knowledge and skills that are articulated in the content standards. At each grade level, the content standards offer natural and productive settings for generating evidence for Claim #2. Tasks generating evidence for Claim #2 in a given grade will draw upon knowledge and skills articulated in the progression of standards up to that grade. Any given task will provide evidence for several of the following assessment targets. Each of the following targets should not lead to a separate task: it is in *using* content from different areas, including work studied in earlier grades, that students demonstrate their problem solving proficiency.
Relevant Verbs for Identifying Content Clusters and/or Standards for Claim #2 “understand” (often in conjunction with one or more other relevant verbs), “solve,” “apply,” “describe,” “illustrate,” “interpret,” and “analyze.”
Target A: Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace. (DOK 2, 3) Under Claim #2, the problems should be completely formulated, and students should be asked to find a solution path from among their readily available tools. (See example "A" below.) **Target B: Select and use appropriate tools strategically.** Tasks used to assess this target should allow students to find and choose tools; for example, using a “Search” feature to call up a formula (as opposed to including the formula in the item stem) or using a protractor in physical space. (DOK 1, 2).

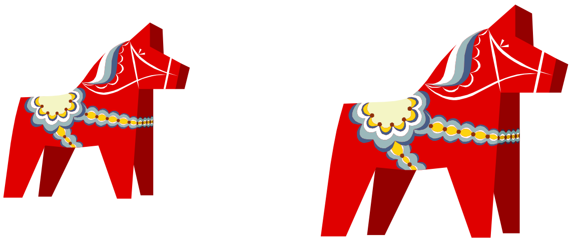
Target C: Interpret results in the context of a situation. (DOK 2)
Tasks used to assess this target should ask students to link their answer(s) back to the problem's context. In early grades, this might include a judgment by the student of whether to express an answer to a division problem using a remainder or not based on the problem's context. In later grades, this might include a rationalization for the domain of a function being limited to positive integers based on a problem's context (e.g., understanding that the negative values for the independent variable in a quadratic function modeling a basketball shot have no meaning in this context, or that the number of buses required for a given situation cannot be 32 1/3[23]).

Target D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). (DOK 1, 2, 3)
For Claim #2 tasks, this may be a separate target of assessment explicitly asking students to use one or more potential mappings to understand the relationship between quantities. In some cases, item stems might suggest ways of mapping relationships to scaffold a problem for Claim #2 evidence.

[23] See, e.g., National Assessment of Educational Progress. (1983). *The third national mathematics assessment: Results, trends, and issues (Report No. 13-MA-01).* Denver, CO: Educational Commission of the States.

An Example Short Answer Item for Claim #2

"Toys for Charity" **(First-year Algebra)**



Phil and Cathy want to raise money for charity.
They decide to make and sell wooden toys.
They could make them in two sizes: small and large.

Phil will carve them from wood.
A small toy takes 2 hours to carve and a large toy takes 3 hours to carve.
Phil only has a total of 24 hours available for carving.

Cath will decorate them.
She only has time to decorate 10 toys.

The small toy will make $8 for charity.
The large toy will make $10 for charity.

They want to make as much money for charity as they can.

How many small and large toys should they make?

How much money will they then make for charity?

For the above example, supporting scaffolding could prompt the student to think about questions like:

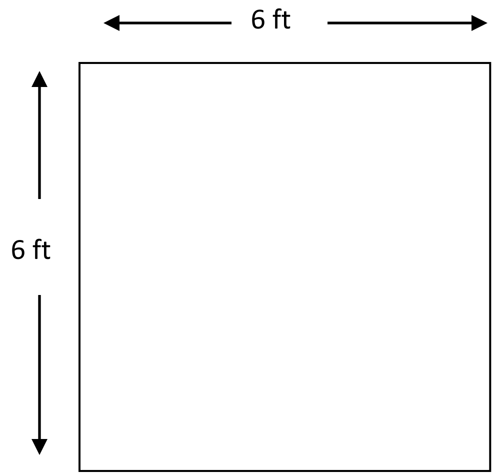
1. If they were to make only small toys, how much money would they make for charity?
2. If they were to make 2 small toys, how many large ones could they also make?

Types of Extended Response Tasks for Claim #2

Problems in pure mathematics: These are well-posed problems within mathematics where the student must find an approach, choose which mathematical tools to use, carry the solution through, and explain the results. For example, students who have access to a graphing calculator can work problems such as the following:

Making a Water Tank

A square metal sheet (6 feet x 6 feet) is to be made into an open-topped water tank by cutting squares from the four corners of the sheet, and bending the four remaining rectangular pieces up, to form the sides of the tank. These edges will then be welded together.



A. How will the final volume of the tank depend upon the size of the squares cut from the corners?

Describe your answer by:

i) Sketching a rough graph

ii) explaining the shape of your graph in words

iii) writing an algebraic formula for the volume

B. How large should the four corners be cut, so that the resulting volume of the tank is as large as possible?

Design problems: These problems have much the same properties but within a design context from the real, or a fantasy, world. See, for example, "sports bag" from the assessment sampler.

Planning problems: Planning problems (like "toys for charity" above) involve the coordinated analysis of time, space, cost – and people. They are design tasks with a time dimension added. Well-posed problems of this kind assess the student's ability to make the connections needed between different parts of mathematics.

This is not a complete list; other types of task that fit the criteria above may well be included. But a balanced mixture of these types will provide enough evidence for Claim #2, as well as contributing evidence with regard to Claim #1. Illustrative examples of each type are shown in the sample items and tasks in Appendix C.

Mathematics Claim #3
COMMUNICATING REASONING

Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

Rationale for Claim #3

This claim refers to a recurring theme in the CCSSM content and practice standards, the ability to construct and present a clear, logical, convincing argument. For older students this may take the form of a rigorous deductive proof based on clearly stated axioms. For younger students this will involve more informal justifications. Assessment tasks that address this claim will typically present a claim and ask students to provide, for example, a justification or counter-example.

Rigor is about precision in argument: first avoiding making false statements, then saying more precisely what one assumes, and providing the sequence of deductions one makes on this basis. Assessments should also include tasks that examine a student's ability to analyze a provided explanation, identify flaws, and correct them.

> "Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments." (Practice 3, CCSSM)

Assessments should include items and tasks that test a student's proficiency in using concepts and definitions in their explanations:

> Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are

careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions. (Practice 6, CCSSM)

What sufficient evidence looks like for Claim #3

Assessment of this claim can be accomplished with a variety of item/task types, including selected response and short constructed response items, and with extended constructed response tasks. Sufficient evidence would be unlikely to be produced if students were not expected to produce communications about their own reasoning and the reasoning of others. That said, students are likely to be unfamiliar with assessment tasks asking them to explain their reasoning. It will be important for early piloting of performance tasks to present these expectations to students in a variety of ways to ensure that the assessment system can develop items and tasks students are able to respond to with success. As students (and teachers) become more familiar with the expectations of the assessment, and as instruction in the Common Core takes hold, students will become more and more successful on tasks aligned to Claim #3 with increasing frequency.

Items and tasks aligned to this claim should reflect the values set out for this claim, giving substantial weight to the quality and precision of the reasoning reflected in at least one, or several of the manners listed below. Options for selected response items and scoring guides for constructed response tasks should be developed to provide credit for demonstration of reasoning and to capture and identify flaws in students' logic or reasoning. Features of options and scoring guides include:

- Assuring an explanation of the assumptions made;
- Asking for or recognizing the construction of conjectures that appear plausible, where appropriate;
- Having the student construct examples (or asking the student to distinguish among appropriate and inappropriate examples) in order to evaluate the proposition or conjecture;
- Requiring the student to describe or identify flaws or gaps in an argument;
- Evaluating the clarity and precision with which the student constructs a logical sequence of steps to show how the assumptions lead to the acceptance or refutation of a proposition or conjecture;
- Rating the precision with which the student describes the domain of validity of the proposition or conjecture.

The set of Claim #3 tasks may involve more than one domain. Because of the high strategic demand that substantial non-routine tasks present, the technical demand will be lower – typically met by content first taught in earlier grades, consistent with the emphases described under Claim #1.

Accessibility and Claim #3: Successful performance under Claim #3 requires a high level of linguistic proficiency. Many students with disabilities have difficulty with written expression, whether via putting pencil to paper or fingers to computer. The claim does not suggest that correct spelling or punctuation is a critical part of the construction of a viable argument, nor does it suggest that the argument has to be in words. Thus, for those students whose disabilities create barriers to development of text for demonstrating reasoning and formation of an argument, it is appropriate to model an argument via symbols, geometric shapes, or calculator or computer graphic programs. As for Claims #1 and #2, access via text to speech and expression via scribe, computer, or speech to text technology will be important avenues for enabling many students with disabilities to construct viable arguments.

The extensive communication skills anticipated by this claim may also be challenging for many ELL students who nonetheless have mastered the content. Thus it will be important to provide multiple opportunities to ELL students for explaining their ideas through different methods and at different levels of linguistic complexity. Based on the data on ELL students' level of proficiency in L1 and L2, it will be useful to provide opportunities as appropriate for bilingual explanations of the outcomes. Furthermore, students' engagement in critique and debate should not be limited to oral or written words, but can be demonstrated through diagrams, tables, and structured mathematical responses where students provide examples or counter-examples of additional problems.

Assessment Targets for Claim #3

Claim #3 is aligned to the mathematical practices from the MCCSS, which are consistent across grade levels. For this reason, the Assessment Targets are not divided into a grade-by-grade description. Rather, a general set of targets is provided, which can be used as guidance for the development of item and test specifications for each grade.

SUMMATIVE ASSESSMENT TARGETS **Providing Evidence Supporting Claim #3**
Claim #3: Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
To preserve the focus and coherence of the standards as a whole, tasks must draw clearly on knowledge and skills that are articulated in the content standards. At each grade level, the content standards offer natural and productive settings for generating evidence for Claim #3. Tasks generating evidence for Claim #3 in a given grade will draw upon knowledge and skills articulated in the standards in that same grade, with strong emphasis on the major work of the grade. Any given task will provide evidence for several of the following assessment targets; each of the following targets should not lead to a separate task.
Relevant Verbs for Identifying Content Clusters and/or Standards for Claim #3 "understand," "explain," "justify," "prove," "derive," "assess," "illustrate," and "analyze."

Target A: Test propositions or conjectures with specific examples. (DOK 2)
Tasks used to assess this target should ask for specific examples to support or refute a proposition or conjecture. (e.g., An item stem might begin, "Provide 3 examples to show why/how…")

Target B: Construct, autonomously,[24] chains of reasoning that will justify or refute propositions or conjectures. (DOK 3, 4).[25]
Tasks used to assess this target should ask students to develop a chain of reasoning to justify or refute a conjecture. Tasks for Target B might include the types of examples called for in Target A as part of this reasoning, but should do so with a lesser degree of scaffolding than tasks that assess Target A alone. (See Example C below. A slight modification of that task asking the student to provide two prices to show Max is incorrect would take away the "autonomous reasoning" requirement necessary for a task to appropriately assess Target B).

Some tasks for this target will ask students to formulate and justify a conjecture.

Target C: State logical assumptions being used. (DOK 2, 3)
Tasks used to assess this target should ask students to use stated assumptions, definitions, and previously established results in developing their reasoning. In some cases, the task may require students to provide missing information by researching or providing a reasoned estimate.

Target D: Use the technique of breaking an argument into cases. (DOK 2, 3)
Tasks used to assess this target should ask students to determine under what conditions an argument is true, to determine under what conditions an argument is not true, or both.

Target E: Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in the argument—explain what it is. (DOK 2, 3, 4)
Tasks used to assess this target present students with one or more flawed arguments and ask students to choose which (if any) is correct, explain the flaws in reasoning, and/or correct flawed reasoning.

Target F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions. (DOK 2, 3)
In earlier grades, the desired student response might be in the form of concrete referents. In later grades, concrete referents will often support generalizations as part of the justification rather than constituting the entire expected response.

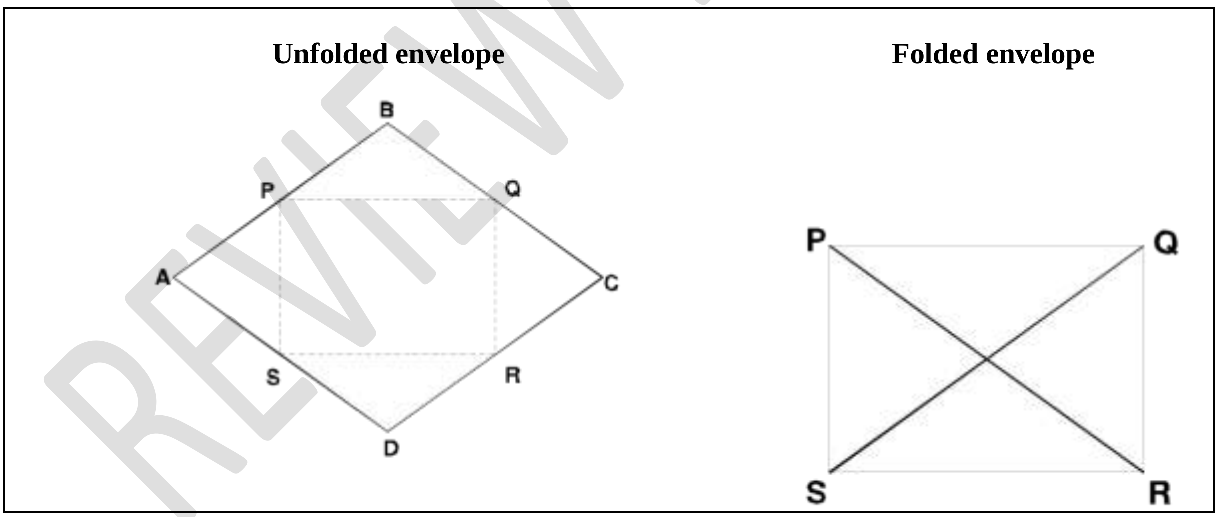
Target G: At later grades, determine conditions under which an argument does and does not apply. (For example, area increases with perimeter for squares, but not for all plane figures.) (DOK 3, 4)
Tasks used to assess this target will ask students to determine whether a proposition or conjecture always applies, sometimes applies, or never applies and provide justification to support their conclusions. Targets A and B will likely be included also in tasks that collect evidence for Target G.

[24] By "autonomous" we mean that the student responds to a single prompt, without further guidance within the task.
[25] At the secondary level, these chains may take a successful student 10 minutes to construct and explain. Times will be somewhat shorter for younger students, but still giving them time to think and explain. For a minority of these tasks, subtasks may be constructed to facilitate entry and assess student progress towards expertise. Even for such "apprentice tasks" part of the task will involve a chain of autonomous reasoning that takes at least 5 minutes.

Types of Extended Response Tasks for Claim #3

Proof and justification tasks: These begin with a proposition and the task is to provide a reasoned argument why the proposition is or is not true. In other tasks, students may be asked to characterize the domain for which the proposition is true (see Assessment Target G).

Example of a standard proof task

Math – Grade 11	**Item Type: CR**	**DOK: (Webb 1- 4) 3**
Domain(s): Geometry **Content Cluster(s) and/or Standard(s):** G.CO **Prove** geometric theorems G.CO.11 **Prove** theorems about parallelograms.		
Claim #3 Assessment Targets Target B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. Target C: State logical assumptions being used. Target F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions.		
The Envelope  Prove that when the rectangular envelope (PQRS) is unfolded, the shape obtained (ABCD) is a rhombus.		

Critiquing tasks: Some flawed 'student' reasoning is presented and the task is to correct and improve it. See, for example, part 2 of task CR2 ("25% sale") in the assessment sampler.

Math – Grade 7	**Item Type: CR**	**DOK: (Webb 1- 4) 3**
Domain(s): Ratios and Proportional Relationships **Content Cluster(s) and/or Standard(s)** 7.RP **Analyze** proportional relationships and use them to **solve** real-world and mathematical problems. 7.RP.3 Use proportional relationships to **solve** multistep ratio and percent problems.		
Claim #3 Assessment Targets Target A: Test propositions or conjectures with specific examples. Target B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. Target D: Use the technique of breaking an argument into cases. Target E: Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in the argument, explain what it is.		
Sale prices Max bought 2 items in a sale. One item was 10% off. One item was 20% off. Max says he saved 15% altogether. Is he right? Explain. 		

Mathematical investigations: Students are presented with a phenomenon and are invited to formulate conjectures about it. They are then asked to go on and prove one of their conjectures. This kind of task benefits from a longer time scale, and might best be incorporated into assessments associated with the Performance Tasks that afford a longer period of time for students to complete their work.

Sums of Consecutive Numbers

Many whole numbers can be expressed as the sum of two or more positive <u>consecutive</u> whole numbers, some of them in more than one way.

For example, the number 5 can be written as

5 = 2 + 3

and that's the only way it can be written as a sum of consecutive whole numbers.

In contrast, the number 15 can be written as the sum of consecutive whole numbers in three different ways:

15 = 7 + 8

15 = 4 + 5 + 6

15 = 1 + 2 + 3 + 4 + 5

Now look at other numbers and find out all you can about writing them as sums of consecutive whole numbers.

Write an account of your investigation. If you find any patterns in your results, be sure to point them out, and also try to explain them fully.

This is not a complete list; other types of task that fit the criteria above may well be included. But a balanced mixture of these types will provide enough evidence for Claim #3. Illustrative examples of each type are given in the sample items and tasks in Appendix C.

Mathematics Claim #4
MODELING AND DATA ANALYSIS

Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

Rationale for Claim #4

Modeling is the bridge across the “school math”/”real world” divide that has been missing from many mathematics curricula and assessments[26]. It is the twin of *mathematical literacy*, the focus of the PISA international comparison tests in mathematics. CCSSM features modeling as both a mathematical practice at all grades and a content focus in high school.

> Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decision-making. (p.72, CCSSM)
>
> Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. (Practice 4; CCSSM)

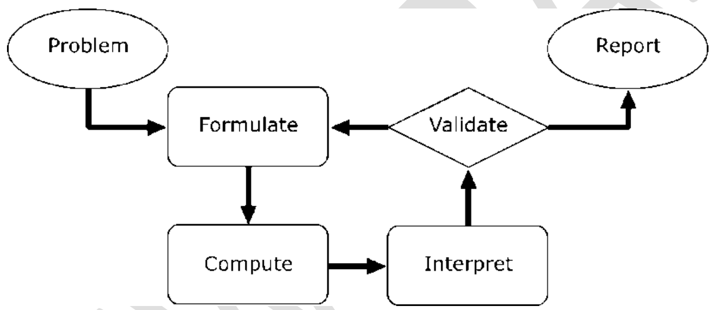
In the real world, problems do not come neatly ‘packaged’. Real world problems are complex, and often contain insufficient or superfluous data. Assessment tasks will involve *formulating* a problem that is tractable using mathematics - that is, formulating a model. This will usually involve making assumptions and simplifications. Students will need to select from the data at hand, or estimate data that are missing. (Such tasks are therefore distinct from the problem solving tasks described in Claim #2, that are well-formulated). Students will identify variables in a situation, and construct relationships between these. When students have formulated the problem, they then tackle it, often in a decontextualized form, before

[26] In their everyday life and work, most adults use none of the mathematics they are first taught after age 11. They often do not see the mathematics that they do use (in planning, personal accounting, design, thinking about political issues etc.) as mathematics.

interpreting their results and checking them for reasonableness.

> Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. (Practice 2; CCSSM)

Finally, students interpret, validate and report their solutions through the successive phases of the modeling cycle, illustrated in the following diagram from CCSSM.



Assessment tasks will also test whether students are able to use technology in this process.

> When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts. (Practice 5; CCSSM)

What sufficient evidence looks like for Claim #4

A key feature of items and tasks in Claim #4 is the student is confronted with a contextualized, or "real world" situation and must decide which information is relevant and how to represent it. As some of the examples provided below illustrate, "real world" situations do not necessarily mean questions that a student might really face; it means that mathematical problems are embedded in a practical, application context. In this way, items and tasks in Claim #4 differ from those in Claim #2, because while the goal is clear, the problems themselves are not yet fully formulated (well-posed) in mathematical terms.

Items/tasks in Claim #4 assess student expertise in choosing appropriate content and using it effectively in formulating models of the situations presented and making appropriate inferences from them. Claim #4 items and tasks should sample across the content domains, with many of these involving more than one domain. Items and tasks of this sort require students to apply mathematical concepts at a significantly deeper level of understanding of mathematical content than is expected by Claim #1. Because of the high strategic demand that substantial non-routine tasks present, the technical demand will be lower – normally met by content first taught in earlier grades, consistent with the emphases described under Claim #1. Although most situations faced by students will be embedded in longer performance tasks, within those tasks, some selected response and short constructed response items will be appropriate to use.

Accessibility and Claim #4: Many students with disabilities can analyze and create increasingly complex models of real world phenomena but have difficulty communicating their knowledge and skills in these areas. Successful adults with disabilities rely on alternative ways to express their knowledge and skills, including the use of assistive technology to construct shapes or develop explanations via speech to text. Others rely on calculators, physical objects, or tools for constructing shapes to work through their analysis and reasoning process.

For English learners, it will be important to recognize ELL students' linguistic background and level of proficiency in English in assigning tasks and to allow explanations that include diagrams, tables, graphic representations, and other mathematical representations in addition to text. It will also be important to include in the scoring process a discussion of ways to resolve issues concerning linguistic and cultural factors when interpreting responses.

Assessment Targets for Claim #4

Claim #4 is aligned to the mathematical practices from the MCCSS, which are consistent across grade levels. For this reason, the Assessment Targets are not divided into a grade-by-grade description. Rather, a general set of targets is provided, which can be used as guidance for the development of item and test specifications for each grade.

SUMMATIVE ASSESSMENT TARGETS Providing Evidence Supporting Claim #4
Claim #4 - Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.
To preserve the focus and coherence of the standards as a whole, tasks must draw clearly on knowledge and skills that are articulated in the content standards. At each grade level, the content standards offer natural and productive settings for generating evidence for Claim #4. Tasks generating evidence for Claim #4 in a given grade will draw upon knowledge and skills articulated in the progression of standards up to that grade, with strong emphasis on the major work of the grades. Any given task will provide evidence for several of the following assessment targets; each of the following

targets should not lead to a separate task.

Relevant Verbs for Identifying Content Clusters and/or Standards for Claim #4

"model," "construct," "compare," "investigate," "build," "interpret," "estimate," "analyze," "summarize," "represent," "solve," "evaluate," "extend," and "apply"

Target A: Apply mathematics to solve problems arising in everyday life, society, and the workplace. (DOK 2, 3)
Problems used to assess this target for Claim #4 should not be completely formulated (as they are for the same target in Claim #2), and require students to extract relevant information from within the problem and find missing information through research or the use of reasoned estimates.

Target B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem. (DOK 2, 3, 4).[27]
Tasks used to assess this target include CR9 ("counting trees") from the assessment sampler, and "design a tent" below.

Target C: State logical assumptions being used. (DOK 1, 2)
Tasks used to assess this target ask students to use stated assumptions, definitions, and previously established results in developing their reasoning. In some cases, the task may require students to provide missing information by researching or providing a reasoned estimate.

Target D: Interpret results in the context of a situation. (DOK 2, 3)
Tasks used to assess this target should ask students to link their answer(s) back to the problem's context. (See Claim #2, Target C for further explication.)

Target E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon. (DOK 3, 4)
Tasks used to assess this target ask students to investigate the efficacy of existing models (e.g., develop a way to analyze the claim that a child's height at age 2 doubled equals his/her adult height) and suggest improvements using their own or provided data.

Other tasks for this target will ask students to develop a model for a particular phenomenon (e.g., analyze the rate of global ice melt over the past several decades and predict what this rate might be in the future). Longer constructed response items and extended performance tasks should be used to assess this target.

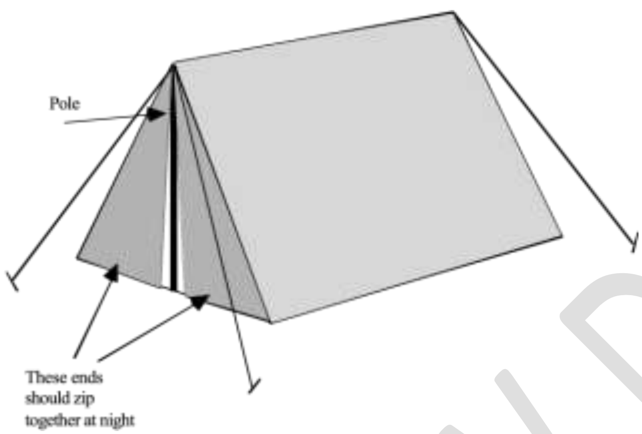
Target F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). (DOK 1, 2, 3)
Unlike Claim #2 where this target might appear as a separate target of assessment (see Claim #2, Target D), it will be embedded in a larger context for items/tasks in Claim #4. The mapping of relationships should be part of the problem posing and solving related to Claim #4 Targets A, B, E, and G.

Target G: Identify, analyze and synthesize relevant external resources to pose or solve problems. (DOK 3, 4)

[27] At the secondary level, these chains should typically take a successful student 10 minutes to complete. Times will be somewhat shorter for younger students, but still giving them time to think and explain. *For a minority of these tasks*, subtasks may be constructed to facilitate entry and assess student progress towards expertise. Even for such "apprentice tasks" part of the task will involve a chain of autonomous reasoning that takes at least 5 minutes.

Especially in extended performance tasks (those requiring 1-2 class periods to complete), students should have access to external resources to support their work in posing and solving problems (e.g., finding or constructing a set of data or information to answer a particular question or looking up measurements of a structure to increase precision in an estimate for a scale drawing). Constructed response items should incorporate "hyperlinked" information to provide additional detail (both relevant and extraneous) for solving problems in Claim #4.

Design a Tent (Grade 8)



Your task is to design a 2-person tent like the one in the picture.

Your design must satisfy these conditions:

- It must be big enough for someone to move around in while kneeling down, and big enough for all their stuff.
- The bottom of the tent will be made from a thick rectangle of plastic.
- The sloping sides and the two ends will be made from a single, large sheet of material.
- Two vertical tent poles will hold the whole tent up.

Make drawings to show how you will cut the plastic and the material.

Make sure you show the measures of all relevant lengths and angles clearly on your drawings, and explain why you have made the choices you have made.

***The Taxicab Problem* (Grade 9)**

You work for a business that has been using two taxicab companies, Company A and Company B.

Your boss gives you a list of (early and late) "Arrival times" for taxicabs from both companies over the past month.

Your job is to analyze those data using charts, diagrams, graphs, or whatever seems best. You are to:

1. Make the best argument that you can in favor of Company A;

2. Make the best argument that you can in favor of Company B;

3. Write a memorandum to your boss that makes a reasoned case for choosing one company or the other, using the relevant mathematical tools at your disposal.

Here are the data:

Company A		Company B	
3 min. 30 sec. EARLY	2 min. 15 sec. LATE	3 min. 45 sec. LATE	1 min. 30 sec. LATE
45 sec. LATE	9 min. 15 sec. LATE	4 min. 30 sec. LATE	3 min. 30 sec. LATE
1 min. 30 sec. LATE	3 min. 30 sec. LATE	3 min. LATE	6 min. LATE
4 min. 30 sec. LATE	1 min. 15 sec. LATE	5 min. LATE	4 min. 30 sec. LATE
45 sec. EARLY	30 sec. EARLY	2 min. 15 sec. LATE	5 min. 30 sec. LATE
2 min. 30 sec. EARLY	2 min. 30 sec. LATE	2 min. 30 sec. LATE	2 min. 30 sec. LATE
4 min. 45 sec. LATE	30 sec. LATE	1 min. 15 sec. LATE	4 min. 15 sec. LATE
3 min. 45 sec. LATE	7 min. 15 sec. LATE	45 sec. LATE	2 min. 45 sec. LATE
30 sec. LATE	5 min. 30 sec. LATE	3 min. LATE	3 min. 45 sec. LATE
1 min. 30 sec. EARLY	3 min. LATE	30 sec. EARLY	4 min. 45 sec. LATE

To work this problem the student needs to decide how to conceptualize the data, which computations to make, and how to represent the data from those computations. It turns out that Company A has a better mean arrival time than company B (this is the core of the argument they should make if they decide in favor of A - and for which they would receive credit), but it has a much greater spread of arrival times. The narrow spread is the compelling argument for B - you can’t risk waiting for a cab that is extremely late, even if the company’s average is good. Thus the best solution is to use company B, but to ask that they come a bit earlier than you actually need them - thus guaranteeing they arrive on time.[28]

With such problems, we see how students decide which information is a given problem context is important, and then how they use it. This is a dimension that is not found in Claim #2.

[28] This problem has been used with thousands of students, and is well within their capacity. It is very different from a problem that gives the students the same numbers and asks them to calculate the mean times, ranges, etc.

Types of Extended Response Tasks for Claim #4

The following types of tasks, when well-designed and developed through piloting, naturally produce evidence on the aspects of a student's performance relevant to this claim. Some examples of are given below, with an analysis of what they assess.

Making decisions from data: These tasks require students to select from a data source, analyze the data and draw reasonable conclusions from it. This will often result in an *evaluation or recommendation*. The purpose of these tasks is not to provide a setting for the student to demonstrate a particular data analysis skill (e.g. box-and-whisker plots)—rather, the purpose is the drawing of conclusions in a realistic setting, using a range of techniques.

Making reasoned estimates: These tasks require students to make reasonable estimates of things they do know, so that they can then build a chain of reasoning that gives them an estimate of something they *do not know*.

<table>
<tr><td>Math – Grade 7</td><td>Item Type: CR</td><td>DOK: (Webb 1- 4) 3</td></tr>
<tr><td colspan="3">Domain(s): Geometry

Content Cluster(s) and/or Standard(s)

7.G Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.SP Investigate patterns of association in bivariate data.</td></tr>
<tr><td colspan="3">Claim #4 Assessment Targets

Target A: Apply mathematics to solve problems arising in everyday life, society, and the workplace.
Target C: State logical assumptions being used.
Target D: Interpret results in the context of a situation.</td></tr>
</table>

Wrap the Mummy

Pam is thirteen today.
She is holding a party at which she plans to play the game 'Wrap the mummy'.
In this game, players try to completely cover themselves with toilet paper.

A roll of toilet paper contains 100 feet of paper, 4 inches wide.

Will one toilet roll be enough to wrap a person?

Describe your reasoning as fully as possible.
(You will need to estimate the average size of an adult person)

Plan and design tasks: Students recognize that this is a problem situation that arises in life and work. Well-posed planning tasks involving the coordinated analysis of time, space, and cost have already been commended for assessing Claim #2. For Claim #4, the problem will be presented in a more open form, asking the student to identify the variables that need to be taken into account, and the information they will need to find. An example of a relatively complex plan and design task is:

Planning a Class Trip

You and your friends on the Class Activities Committee are charged with deciding where this year's class trip will be. You have a fixed budget for the class and you need to figure out what will be the most fun and affordable option. Your committee members have collected a bunch of brochures from various parks - e.g., Marine World, Great Adventure, and others (see inbox of materials) - which have different admissions costs and are different distances from school. You have also collected information about the costs of meals and buses. Your job is to plan and justify a trip that includes bus fare, admission and possibly rides, as well as lunch, within the fixed budget the class has.

Evaluate and recommend tasks: These tasks involve understanding a model of a situation and/or some data about it and making a recommendation. For example:

Safe driving distances

A car with good brakes can stop in a distance "*D*" feet that is related to its speed "*v*" miles per hour by the model:

$$D = 1.5vt + v^2/20$$

where "*t*" is the driver's reaction time in seconds.

Using this model, you have been asked to recommend how close behind the car ahead it is safe to drive (in feet) for various speeds of *v* miles per hour.

Interpret and critique tasks: These tasks involve interpreting some data and critiquing an argument based on it. Again the purpose of these tasks is not to provide a setting for the student to demonstrate a particular data analysis skill, but to draw conclusions in a realistic setting, using a range of techniques. For example:

Choosing for the Regionals

Our school has to select a girl for the long jump at the regional championship. Three girls are in contention. We have a school jump-off. Their results, in meters, are given below:

Elsa	Ilse	Olga
3.25	3.55	3.67
3.95	3.88	3.78
4.28	3.61	3.92
2.95	3.97	3.62
3.66	3.75	3.85
3.81	3.59	3.73

Hans says, "Olga has the longest average. She should go to the championship."

Do you think Hans is right? Is Olga the best choice? Explain your reasoning.

This is not a complete list; other types of task that fit the criteria above may well be included. A balanced mixture of these types will provide enough evidence for Claim #4.

References

(Complete citations to be added in final version)

Van Hiele, Pierre (1985) [1959], *The Child's Thought and Geometry*, Brooklyn, NY: City University of New York, pp. 243-252

Appendix A – Grade-Level Content Emphases

The tables on the following pages summarize the cluster-level emphases (major, additional, and supporting) for grades 3-8 and Grade 11.

Grade 3 Cluster-Level Emphases

m = major clusters; a = additional clusters; s = supporting clusters

Operations and Algebraic Thinking

[m]: Represent and solve problems involving multiplication and division.

[m]: Understand properties of multiplication and the relationship between multiplication and division.

[m]: Multiply and divide within 100.

[m]: Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Number and Operations in Base Ten

[a]: Use place value understanding and properties of arithmetic to perform multi-digit arithmetic. (DOK 1)

Number and Operations—Fractions

[m]: Develop understanding of fractions as numbers. (DOK 1, 2)

Measurement and Data

[m]: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. (DOK 1, 2)

[s]: Represent and interpret data. (DOK 2, 3)

[m]: Geometric measurement: understand concepts of area and relate area to multiplication and to addition. (DOK 1, 2)

[a]: Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures. (DOK 1)

Geometry

[s]: Reason with shapes and their attributes. (DOK 1, 2)

Mathematical Practices summary

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 4 Cluster-Level Emphases

m = major clusters; a = additional clusters; s = supporting clusters

Operations and Algebraic Thinking

[m] Use the four operations with whole numbers to solve problems.

[s] Gain familiarity with factors and multiples.

[a] Generate and analyze patterns.

Number and Operations in Base Ten

[m] Generalize place value understanding for multi-digit whole numbers.

[m] Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions

[m] Extend understanding of fraction equivalence and ordering.

[m] Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

[m] Understand decimal notation for fractions, and compare decimal fractions.

Measurement and Data

[s] Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

[s] Represent and interpret data.

[a] Geometric measurement: understand concepts of angle and measure angles.

Geometry

[a] Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Mathematical Practices summary

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 5 Cluster-Level Emphases

m = major clusters; a = additional clusters; s = supporting clusters

Operations and Algebraic Thinking

[a] Write and interpret numerical expressions.

[a] Analyze patterns and relationships.

Number and Operations in Base Ten

[m] Understand the place value system.

[m] Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations— Fractions

[m] Use equivalent fractions as a strategy to add and subtract fractions.

[m] Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Measurement and Data

[s] Convert like measurement units within a given measurement system.

[s] Represent and interpret data.

[m] Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Geometry

[a] Graph points on the coordinate plane to solve real-world and mathematical problems.

[a]Classify two-dimensional figures into categories based on their properties.

Mathematical Practices summary

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 6 Cluster-Level Emphases

m = major clusters; a = additional clusters; s = supporting clusters

Ratios and Proportional relationships

[m] Understand ratio concepts and use ratio reasoning to solve problems.

The Number System

[m] Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
[a] Compute fluently with multi-digit numbers and find common factors and multiples.
[m] Apply and extend previous understandings of numbers to the system of rational numbers.

Expressions and Equations

[m] Apply and extend previous understandings of arithmetic to algebraic expressions.
[m] Reason about and solve one-variable equations and inequalities.
[m] Represent and analyze quantitative relationships between dependent and independent variables

Geometry

[s] Solve real-world and mathematical problems involving area, surface area, and volume.

Statistics and Probability

[a] Develop understanding of statistical variability.

[a] Summarize and describe distributions.

Mathematical Practices summary

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 7 Cluster-Level Emphases

m = major clusters; a = additional clusters; s = supporting clusters

Ratios and Proportional relationships

[m] Analyze proportional relationships and use them to solve real-world and mathematical problems.

The Number System

[m] Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Expressions and Equations

[m] Use properties of operations to generate equivalent expressions.

[m] Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Geometry

[a] Draw, construct and describe geometrical figures and describe the relationships between them.

[a] Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Statistics and Probability

[s] Use random sampling to draw inferences about a population.

[a] Draw informal comparative inferences about two populations.

[s] Investigate chance processes and develop, use, and evaluate probability models.

Mathematical Practices summary

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 8 Cluster-*Level Emphases*

m = major clusters; a = additional clusters; s = supporting clusters

The Number System

[s] Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and equations

[m] Work with radicals and integer exponents.

[m] Understand the connections between proportional relationships, lines, and linear equations.

[m] Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions

[m] Define, evaluate, and compare functions.

[s] Use functions to model relationships between quantities.

Geometry

[m] Understand congruence and similarity using physical models, transparencies, or geometry software.

[m] Understand and apply the Pythagorean theorem.

[a] Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

Statistics and Probability

[s] Investigate patterns of association in bivariate data.

Mathematical Practices summary

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 11 Emphases

The following aspects of the standards play an especially prominent role in college and career readiness:

- The Standards for Mathematical Practice, viewed in connection with mathematical content. Postsecondary instructors value expertise in fundamentals over broad topic coverage (ACT 2006, 2009).
- Modeling and rich applications (see pages 72 and 73 in the standards), which can be integrated into curriculum, instruction and assessment.
  - Note the star symbols («) in the high school Standards for Mathematical Content, which identify natural opportunities to connect the modeling practice to content.
  - Many modeling tasks in high school will require application of content knowledge first gained in grades 6–8 to solve complex problems. (See p. 84 of the standards.)

The following clusters of high school standards have wide relevance as prerequisites for a range of postsecondary college and career pathways:

Number and Quantity: Quantities

Reason quantitatively and use units to solve problems.

Number and Quantity: The Real Number System

Extend the properties of exponents to rational exponents.

Use properties of rational and irrational numbers.

Algebra: Seeing Structure in Expressions

Interpret the structure of expressions.

Write expressions in equivalent forms to solve problems.

Algebra: Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials.

Algebra: Creating Equations

Create equations that describe numbers or relationships.

Algebra: Reasoning with Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning.

Solve equations and inequalities in one variable.

Represent and solve equations and inequalities graphically.

Functions: Interpreting Functions

Understand the concept of a function and use function notation.

Analyze functions using different representations.

Interpret functions that arise in applications in terms of a context.

Functions: Building Functions

Build a function that models a relationship between two quantities.

Geometry: Congruence

Prove geometric theorems.

Statistics and Probability: Interpreting Categorical and Quantitative Data

Summarize, represent and interpret data on a single count or measurement variable.

Mathematical Practices summary

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Appendix B – Cognitive Rigor Matrix/Depth of Knowledge (DOK)

The Common Core State Standards require high-level cognitive demand, such as asking students to demonstrate deeper conceptual understanding through the application of content knowledge and skills to new situations and sustained tasks. For each Assessment Target in this document, the depth(s) of knowledge (DOK) that the student needs to bring to the item/task has been identified, using the Cognitive Rigor Matrix shown below. This matrix draws from two widely accepted measures to describe cognitive rigor: Bloom's (revised) Taxonomy of Educational Objectives and Webb's Depth-of-Knowledge Levels. The Cognitive Rigor Matrix has been developed to integrate these two models as a strategy for analyzing instruction, for influencing teacher lesson planning, and for designing assessment items and tasks. (To download full article describing the development and uses of the Cognitive Rigor Matrix and other support CRM materials, go to: http://www.nciea.org/publications/cognitiverigorpaper_KH11.pdf)

A "Snapshot" of the Cognitive Rigor Matrix (Hess, Carlock, Jones, & Walkup, 2009)

Depth of Thinking (Webb) + Type of Thinking (Revised Bloom)	**DOK Level 1 Recall & Reproduction**	**DOK Level 2 Basic Skills & Concepts**	**DOK Level 3 Strategic Thinking & Reasoning**	**DOK Level 4 Extended Thinking**
Remember	- Recall conversions, terms, facts			
Understand	-Evaluate an expression -Locate points on a grid or number on number line -Solve a one-step problem -Represent math relationships in words, pictures, or symbols	- Specify, explain relationships -Make basic inferences or logical predictions from data/observations -Use models /diagrams to explain concepts -Make and explain estimates	-Use concepts to solve non-routine problems -Use supporting evidence to justify conjectures, generalize, or connect ideas -Explain reasoning when more than one response is possible -Explain phenomena in terms of concepts	-Relate mathematical concepts to other content areas, other domains -Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
Apply	-Follow simple procedures -Calculate, measure, apply a rule (e.g., rounding) -Apply algorithm or formula -Solve linear equations -Make conversions	-Select a procedure and perform it -Solve routine problem applying multiple concepts or decision points -Retrieve information to solve a problem -Translate between representations	-Design investigation for a specific purpose or research question - Use reasoning, planning, and supporting evidence -Translate between problem & symbolic notation when not a direct translation	-Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze	-Retrieve information from a table or graph to answer a question -Identify a pattern/trend	-Categorize data, figures -Organize, order data -Select appropriate graph and organize & display data -Interpret data from a simple graph -Extend a pattern	-Compare information within or across data sets or texts -Analyze and draw conclusions from data, citing evidence -Generalize a pattern -Interpret data from complex graph	-Analyze multiple sources of evidence or data sets

Evaluate			-Cite evidence and develop a logical argument -Compare/contrast solution methods -Verify reasonableness	-Apply understanding in a novel way, provide argument or justification for the new application
Create	- Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	-Generate conjectures or hypotheses based on observations or prior knowledge and experience	-Develop an alternative solution -Synthesize information within one data set	-Synthesize information across multiple sources or data sets -Design a model to inform and solve a practical or abstract situation

Appendix C – Grade 8 Assessment Sampler

This collection provides examples of the kinds of items and tasks that could be found on an assessment for grade 8. The items and tasks shown here represent a variety of types of questions that tap a range of the grade 7 and 8 Common Core State Standards. As noted in the Content Specifications document, when asked to apply knowledge in contexts demonstrating more sophisticated mathematical practices, students will often use some of the content learned in prior grade levels.

Although this collection of tasks reflects the focus and coverage that would be appropriate to represent the standards, it should not be viewed as a sample assessment, as the purpose of this document is not to provide a sample, or practice test. Rather, the purpose here is to provide users with a glimpse as to the mathematical knowledge and skills students will be expected to demonstrate and the ways in which they could be called upon to demonstrate their understanding.

The examples are divided into three parts. Part I contains a collection of "Short Items", demonstrating the kinds of items that might be used solely for Claim #1. Following each of these short items we identify the content standard and claim addressed by that item.

Part II contains a series of "Constructed Response Tasks" of the type that might be used to assess other Claims. Part IIa includes computer-implemented constructed response task sequences that illustrate ways in which a complex task can be structured as a sequence of short computer-based constructed response items that focus on the same content area. Part IIb includes more complex tasks requiring longer chains of reasoning that ask students to integrate mathematical practices and content. Each task in Part IIb is followed by a discussion of the standards, practices, and claims addressed in the task. Also included are elements that would be used to construct a scoring rubric.

Part III contains a single example of an "Extended Performance Task", which would represent the kind of classroom-based task that students might need to work on across more than one day.

Sources for all of the tasks are given at the end of this document. The items and tasks in this document have not been subjected to review/revision procedures that will be part of item and task development for all items/tasks used in the SMARTER Balanced assessments. Review/revision protocols will include Content Review to assure alignment to the mathematics content standards and to Bias and Sensitivity Review to assure that language complexity and cultural features do not intrude on the assessment of student knowledge and skill of mathematics.

Part I: Short Items

1. Write [or, enter; see the format for task 11] the repeating decimal 0.090909………… as a rational number.

____

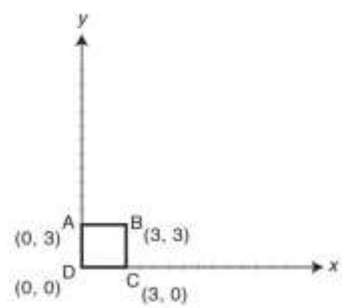
Item 1 addresses Content Standard NS-8.1 and Claim #1

2. If x and y are positive integers, and $3x + 2y = 13$, what could be the value of y? Write [or, enter] all possible answers.

____

Item 2 addresses Content Standard EE-8.1 and Claim #1

3. The diagram below shows four points that are connected to form square ABCD.



Square ABCD will be transformed into quadrilateral A′B′C′D′ using the rule $(x, y) \rightarrow (2x, 3y)$. What type of quadrilateral will image A′B′C′D′ be?

- ○ a square
- ○ a rectangle
- ○ a rhombus

Item 3 addresses Content Standard G-8.2 and Claim #1

4. Which one of the numbers below has the same value as $\mathbf{3.5 \times 10^{-3}}$?

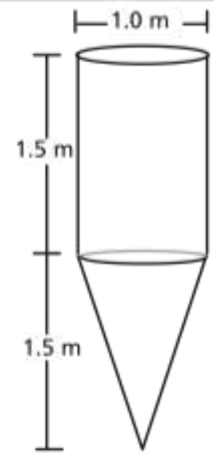
- ○ 35×10^{-4}
- ○ 3.5×10^{3}
- ○ 0.00035
- ○ 3500

Item 4 addresses Content Standard EE-8.1 and Claim #1

5. **Water Tank**

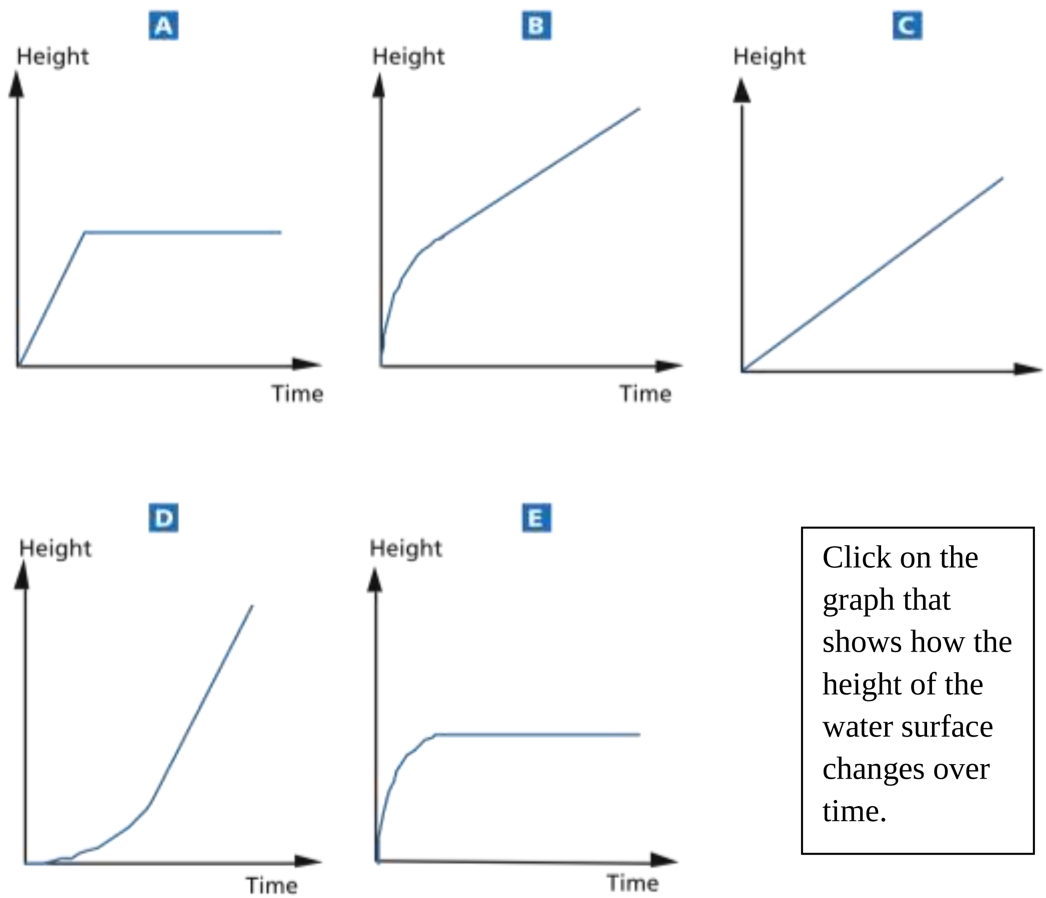
A water tank has shape and dimensions as shown in the diagram.

At the beginning the tank is empty. Then it is filled with water at the rate of one litre per second.



Water tank

Click on the graph that shows how the height of the water surface changes over time.



Click on the graph that shows how the height of the water surface changes over time.

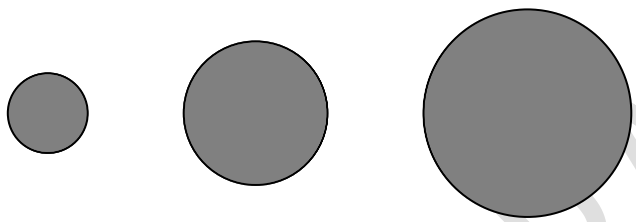
Item 5 addresses Content Standard F-8.5 and Claim #1

6. Jane, Maria, and Ben each have a collection of videos. Jane has 15 more videos than Ben, and Maria has 2 times as many videos as Ben. In all they have 95 videos. How many videos does Maria have? ____________________

Item 6 addresses Content Standard EE-8.7 and Claim #1

7. **Coins**

You are asked to design a new set of coins. All the coins must be circular, and they will be made of the same metal. They will have different diameters, for example



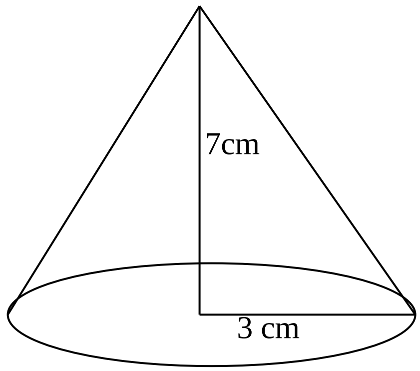
Researchers have decided that the coin system should meet the following requirements:

- the diameter of a coin should not be smaller than 15 mm and not be larger than 45 mm.
- given a coin, the diameter of the next larger coin must be at least 30% larger.
- the machine that makes the coins can only produce coins whose diameter is a whole number of millimeters - so, for example, 17 mm is allowed, but 17.3 mm is not.

You are asked to design a set of coins that meets these requirements. You should start with a 15 mm coin and your set should contain as many coins as possible. Write the diameters of all of the coins in your set.

Item 7 addresses Content Standard RP-7.3 and Claim #1

8. Write [or, enter] the volume of the cone in the figure below.



____

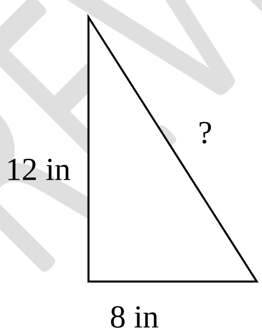
Item 8 addresses Content Standard G-8.9 and Claim #1

9. A cubical block of metal weighs 6.4×10^6 pounds. How much will another cube of the same metal weigh if its sides are half as long?

____

Item 9 addresses Content Standard EE-8.4 and Claim #1

10. If one leg of the right triangle in the figure below is 8 inches long and the other leg is 12 inches long, how many inches long is the triangle's hypotenuse?



____

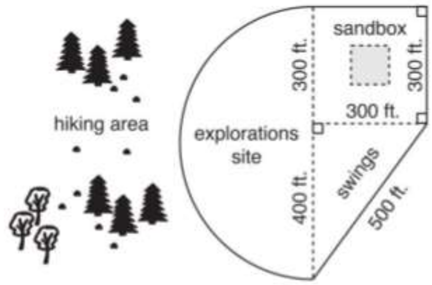
Item 10 addresses Content Standard G-8.7 and Claim #1

Part IIa: Computer-Implemented Constructed Response Task Sequences

Items 11, 12, and 13 illustrate ways in which a complex task can be structured as a sequence of short computer-implemented constructed response items that focus on the same high priority content area.

All City Park

11.



The All City Recreation Committee plans to put a fence around a playground area in All City Park. The solid line in the diagram above outlines the sections in the park that the committee wants to surround with a fence. Information about fencing prices is shown below:

FENCE-ALL COMPANY	ACME FENCE COMPANY
Fencing: $0.30 per foot	Fencing: $0.32 per foot Orders totaling $500 or more will receive a 10% discount.

a. How much fencing will the committee need to buy? Show your work.

Enter response here

Submit

b. Based on the information above, determine which fencing company offers the best deal for this project. Explain your reasoning and show all your work.

Enter response here

Submit

Item 11a addresses Content Standard G-7.4 and Claim #1

Item 11b addresses Content Standard RP-7.3 and Claim #1

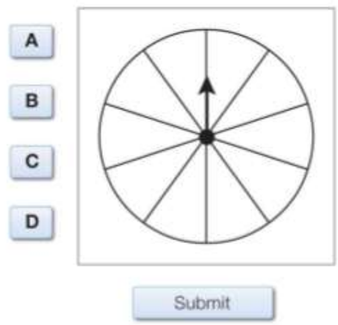
12.

A spinner has 10 sections of equal size. Each section on the spinner is labeled with one letter (A, B, C, or D). The arrow on the spinner was spun 40 times. The results of the spins are recorded in the table below.

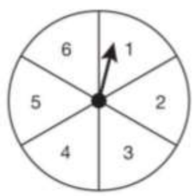
Spinner Results

Letter	Number of Spins
A	8
B	12
C	4
D	16

Based on the data in the table, complete the spinner below to show the number of sections that are most likely labeled with each letter. Click on the letter you want to select. Then click where you would like to place the letter on the spinner.



The spinner below is divided into six equal sections. Each section is marked with a number from 1 to 6.



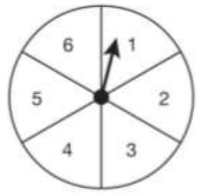
a. The theoretical probability of spinning each number is $\frac{1}{6}$. Based on this probability, how many times should each number occur in 20 spins?

Enter response here Submit

(*continued*)

12 (continued).

You will now conduct an experiment by spinning the spinner 20 times. Use the spinner below to conduct the experiment. Use the spin button to run each trial, then tabulate the results on your scratch paper.



Spin

b. Create a frequency table in the template below that shows the results of the spins. Provide appropriate labels for the table.

Submit

(continued)

c. Explain clearly why your answer from **part a** is different from or the same as the results given in the table above.

Enter response here

Submit

d. If the spinner were spun 200 more times, how would the frequency of the results be affected?

Enter response here

Submit

Item 12a addresses Content Standard SP-7.5 and Claim #1

Item 12b addresses Content Standard SP-7.7 and Claim #1

Item 12c addresses Content Standard SP-7.7 and Claim #1

Item 12d addresses Content Standard SP-7.7 and Claim #1

13.

Content Standards Assessed:

Algebra: Solve systems of linear equations algebraically and graphically, focusing on pairs of linear equations in two variables.

Functions: Interpret the rate of change and constant term of a linear function or sequence in terms of the situation it models, and in terms of its graph or a table of values.

During a football game, the concession stand sells popsicles in five flavors. The manager of the concession stand can buy the popsicles from one of two companies. The chart shows the costs of doing business with each company.

	Company Name	Delivery Cost	Cost per popsicle
Company 1	ABC Food Company	\$80.00	\$1.00
Company 2	Best Price Foods	\$120.00	\$0.75

A Set up a system of linear equations to show the cost to buy popsicles from the two companies.

Company 1	Equation	Company 2	Equation
ABC Food Company	Enter equation here	Best Price Foods	Enter equation here

Submit

B Create a graph to plot the equations.

Give the graph a **title** Enter graph title here

Label for the **x-axis** Enter title for x-axis here

Label for the **y-axis** Enter title for y-axis here

Scale for **x-axis**: Origin value ☐ Interval ☐

Scale for **y-axis** Origin value ☐ Interval ☐

Slope and intercept for ABC Foods Slope ☐ Intercept ☐

Slope and intercept for Best Price Foods Slope ☐ Intercept ☐

Submit

13 (continued).

Check your graph to make sure it represents the two linear equations. If you want to edit the graph, click Edit. After you edit the graph, click on the Submit button again.

When your graph is correct, click Accept.

C Solve the system of equations to determine the values of x and y.

x = ______ y = ______

D What do the values of x and y mean in this context?

Enter response here

E How many popsicles does the concession stand have to sell to make ABC Food Company the best option?

Enter response here

F How many popsicles does the concession stand have to sell to make Best Price Food Company the best option?

Enter response here

Item 13a addresses Content Standard EE-7.4 and Claim #1

Item 13b addresses Content Standard EE-8.8 and Claim #1

Item 13c addresses Content Standard F-8.4 and Claim #1

Item 13d addresses Content Standard F-8.5 and Claim #1

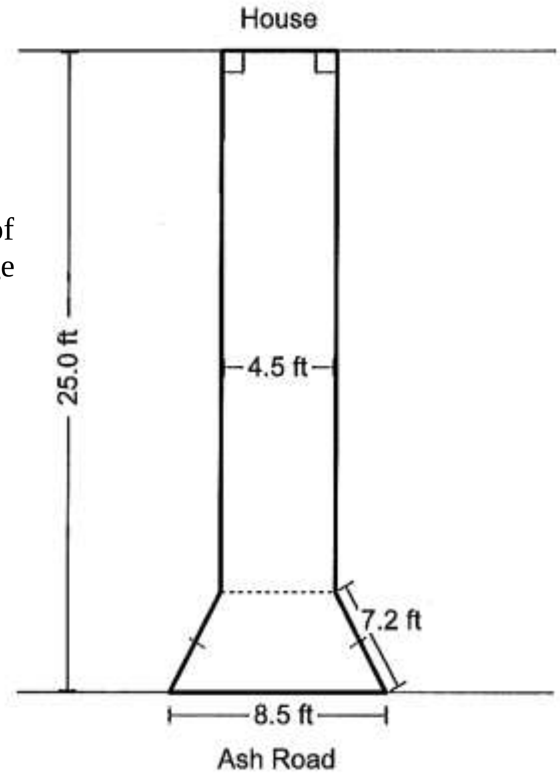
Part IIb: Constructed Response Tasks

These more complex and non-routine tasks ask students to integrate mathematical practices and content, as indicated in the analytic table below. The demands of each task, along with the elements to be considered in a scoring rubric are provided following each task.

Content domains	Ms. Olsen's Sidewalk	25% Sale	Sports Bag	Baseball Jerseys	Jane's TV	The Spinner Game	Bird and Dinosaur Eggs	Taxi Cab	Counting Trees	Shelves	Short Items (cluster#)
Number/Quantity	•	•	•	•	•	•	•	•	•	•	•
Expressions and Equations				•				•			•
Functions				•	•					•	•
Geometry	•		•		•						•
Statistics						•	•				•
Practices											
Make sense / Persevere ...	•		•	•		•		•	•	•	
Reason ….		•				•	•	•			•
Construct/critique						•					
Model			•	•				•			
Use tools		•	•		•		•		•		
Precision	•						•		•		•
Structure			•	•		•				•	
Regularity						•					

CR 1: Ms. Olsen's Sidewalk

Ms. Olsen is having a new house built on Ash Road.
She is designing a sidewalk from Ash Road to her front door.
Ms Olsen wants the sidewalk to have an end in the shape of an isosceles trapezoid, as shown.



The contractor charges a fee of $200 plus $12 per square foot of sidewalk. Based on the diagram, what will the contractor charge Ms. Olsen for her sidewalk?

Show your work or explain how you found your answer.

Discussion

Ms Olsen's Sidewalk addresses:

Content Standards 7.G.6, 7.NS.3, 8.G.7
Practices P1, P5.
Claims 1 and 2.

In this task students are given a real-world problem whose solution involves determining the areas of two-dimensional shapes as part of calculating the cost of a sidewalk. This particular compound shape could be divided in more than one way and a choice needs to be made as to whether the shape should be considered as a rectangle and trapezoid or a longer rectangle with two smaller right-angles triangles appended near Ash Road (these can be thought of as two halves of a rectangle of width 2ft and diagonal 7.2ft). The dashed line leads towards the former.

A common problem with the calculation of the areas of trapezoids is the misuse of the length marked 7.2 ft. Students will need to make use of this dimension but must avoid falling into the error of multiplying 8.5 x 7.2 in an attempt to find the area of the trapezoid. Once the decision has been made regarding how to best deconstruct the figure students will need to apply the Pythagorean Theorem in order to calculate the length of the path contained with the trapezoid.

When this has been calculated the remaining length and area calculations can be undertaken. The final stage of this multi-step problem is to calculate the cost of the paving based on the basic fee of $200 plus $12 per square foot.

This task demands students work across a range of mathematical practices. In particular, they need to:

- ***Make sense of problems and persevere in solving them*** (P1). They will need to analyze the information given and choose a solution pathway.
- ***Attend to precision*** **(**P6) in their careful use of units in the cost calculations.

Rubric Elements

	Ms. Olsen's Sidewalk	Rubric	
		Points	Section points
	Uses the Pythagorean Theorem to find the height of the trapezium. Height = √ (51.84 – 4) = √ 47.84 Finds the correct height of the trapezium = **6.92 = 7 ft** Finds the area of the trapezium = ½(8.5 + 4.5) x 7 = **45.5 ft^2** Finds the area of the rectangle = 81 ft^2 Finds the total area of the sidewalk = 126.5 ft^2 Finds the total charge = $200 + $12 x 126.5 = **$1718**		
	Total Points		

Note:
For scoring purposes, the points for each element can be weighted to reflect the importance of that element relative to the entire task. Further, "Total Points" may be treated in several ways.

One approach is to use "value points", where the total number of points is broken into segments, each accounting for a single score point. For example: if there are a total of 10 points in the rubric, but the task is determined to be valued at 3 points on the test, the rubric may allocate the 10 total points as: 0 value points = Score 0; 1-3 value points = Score 1; 4-7 value points = Score 2; 8-10 value points = Score 3.

An alternate scoring scheme simply awards test points on the basis of features of the task. In the "Ms. Olsen's Sidewalk" task above, if the task is worth 3 points on the test, all 3 points could be awarded if both the final answer for cost and the final square footage are accurate; 2 points could be for only having the square footage is accurate; 1 point for using the Pythagorean Theorem but with an error in calculations; and 0 points for not having any these.

CR2: 25% Sale

In a sale, all the prices are reduced by 25%.

1. Julie sees a jacket that cost $32 before the sale.
   How much does it cost in the sale?

   $ ____________________

   Show your calculations.

In the second week of the sale, the prices are reduced by 25% of the previous week's price.
In the third week of the sale, the prices are again reduced by 25% of the previous week's price.
In the fourth week of the sale, the prices are again reduced by 25% of the previous week's price.

2. Julie thinks this will mean that the prices will be reduced to $0 after the four reductions because

   4 x 25% = 100%.

   Explain why Julie is wrong.

   ____

   ____

   ____

3. If Julie is able to buy her jacket after the four reductions, how much will she have to pay?

$ ______________________

Show your calculations.

Julie buys her jacket after the four reductions.
What percentage of the original price does she save?

____________________%

Show your calculations.

Discussion

25% Sale addresses:

Content Standard 7.RP.3
Practices P2 and P5
Claims 1, 2, and 3

This task assesses students' ability to calculate percentage reductions, compound percentages and reason with percents.

This task is scaffolded with the use of a fairly straightforward percentage calculation in question 1. This might be approached in a single step (75% of 32) or as a two-step calculation (find 25% of $32, then subtract from $32). In order to select the first option students will have a good understanding of 'per cent', that a 25% reduction leaves 75% of the original amount. Those thinking like this will find the final stage of the task more straightforward.

Given the number involved in the problem, students can move between percentage and fractional quantities so rather than working with 75% and 25% they might choose instead to use ¾ and ¼ which in this case works well with a starting quantity of $32.

The task proceeds to explore a common misconception in proportional reasoning, namely that four years of 25% reductions is equivalent to 100% off the original price. Here the students are asked to explain the error in this kind of thinking and this gives good opportunities for assessing students' conceptual understanding of the mathematics. Students might go about this by showing what happens in the particular case introduced in part 1 of the task (32, 24, 18, etc). More ambitious explanations might explain that Julie has confused the notion of percentage or proportion with a fixed amount, that 25% is the same amount regardless of the starting value. This is not any easy explanation to make. They might implicitly refer to the notion of limits, that if you only ever take 25% percent away then there must be something left so the amount cannot reduce to $0. Whatever approach is used there is a really good opportunity here to display mathematical reasoning and argumentation.

Part 3 of the questions formalizes the previous discussion by asking students to calculate the cost of the jacket after four reductions of 25%. For many this will involve a repeated calculation of either 25% (0.25 or ¼) with subtractions from the previous price or, more simply of 75% (3/4) of the previous price. Interestingly, students might change their method as this repeated calculation proceeds. The first two reductions are integers and can easily be calculated using fractions. The last two prices are non-integer and most students will probably make use of a calculator at this stage. The highest attaining students might reduce the calculation to the more elegant single calculation involving multiplications by .75 rather than subtractions.

This task demands that students work across a range of mathematical practices. In particular, they need to:

- ***Reason abstractly and quantitatively*** (P2) in the context of percentage.
- ***Use appropriate tools strategically*** (P5), in this case to calculate percentage reductions accurately.

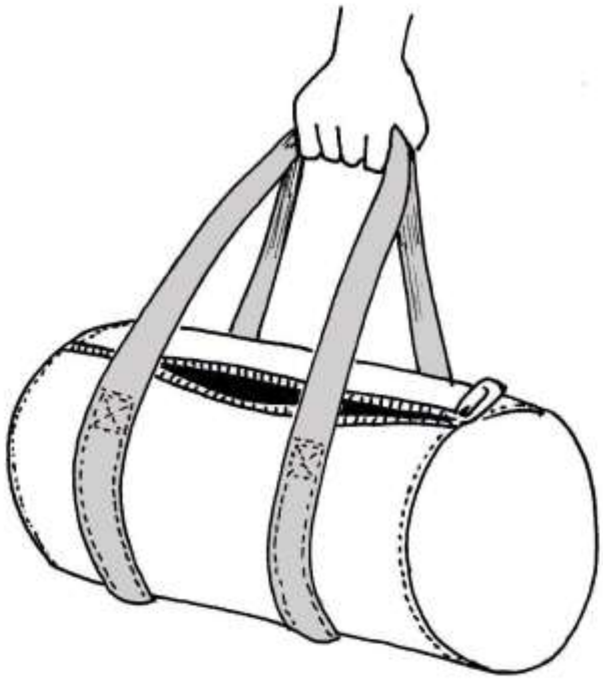
Rubric Elements

25% sale		
		points
1	Gives correct answer: $24 Shows correct work such as: 32 ÷ 4 = 8 and 32 – 8 = 24	
2	Gives a correct explanation such as: Each reduction is 25% of the previous week's price, and as the price goes down each week, the 25% will be smaller amount each week.	
3	Gives correct answer: $10.12 or $10.13 Shows correct work such as: 32×0.75^4 or 24×75^3 or 24 – (24 x 0.25) = 18 18 – (18 x 0.25) = 13.5 13.5 – (13.5 x 0.25) = 10.13 **Partial credit** Correct at least as far as 24 – (24 x 0.25) = 18; 18 – (18 x 0.25) = 13.5 Correct as far as 24 – (24 x 0.25) = 18	
4	Give correct answer: 68.3% or 68.4% Shows correct work such as: 32 – 10.12(or 10.13) = 21.88(21.87) **and** 21.88(21.87) / 32 x 100 = 68.3% **or** 10.12(or 10.13) / 32 x 100 = 21.6(21.7) **and** 100 – 31.6(31.7) = 68.4% **Partial credit** 31.6% or 31.7% with correct work 31.6% or 31.7% without correct work	
	Total Points	

Note:

See scoring comments on the "Ms. Olsen's Sidewalk" task, above

CR 3: Sports Bag



You have been asked to design a sports bag.

- The length of the bag will be 60 cm.
- The bag will have circular ends of diameter 25 cm.
- The main body of the bag will be made from 3 pieces of material; a piece for the curved body, and the two circular end pieces.
- Each piece will need to have an extra 2 cm all around it for a seam, so that the pieces may be stitched together.

1. Make a sketch of the pieces you will need to cut out for the body of the bag.
   Your sketch does not have to be to scale.
   On your sketch, show all the measurements you will need.

2. You are going to make one of these bags from a roll of cloth 1 meter wide. What is the shortest length that you need to cut from the roll for the bag?
   Describe, using words and sketches, how you arrive at your answer.

Discussion

Sports Bag addresses:

Content Standards 7.G.4, 7.G.6
Practices P1, P4, P5, and P6
Claims 1, 2, and 4.

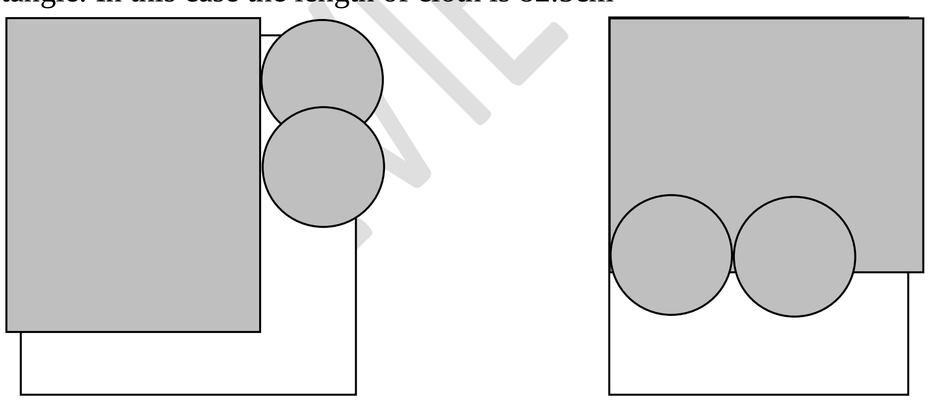
This task involves solving real-world problems involving areas of two-dimensional shapes.

The first task is to recognize that the curved surface of the bag is a rectangle with the length given and the breadth equivalent to the circumference of the circular end of the bag. This observation, along with the observation that one must allow for the extra material around the edges of the shapes, puts the student in a position to make the relevant sketches,

Students are given the diameter of the bag and need to use this to calculate, with the aid of a calculator, the circumference of the circular ends and therefore the missing dimension of the curved surface, which is around 78.5cm.

This means that there will need to be three sketches (two circles and one rectangle) which have the dimensions (including extra) of 29cm diameter circles and a rectangle measuring 64 x 82.5cm.

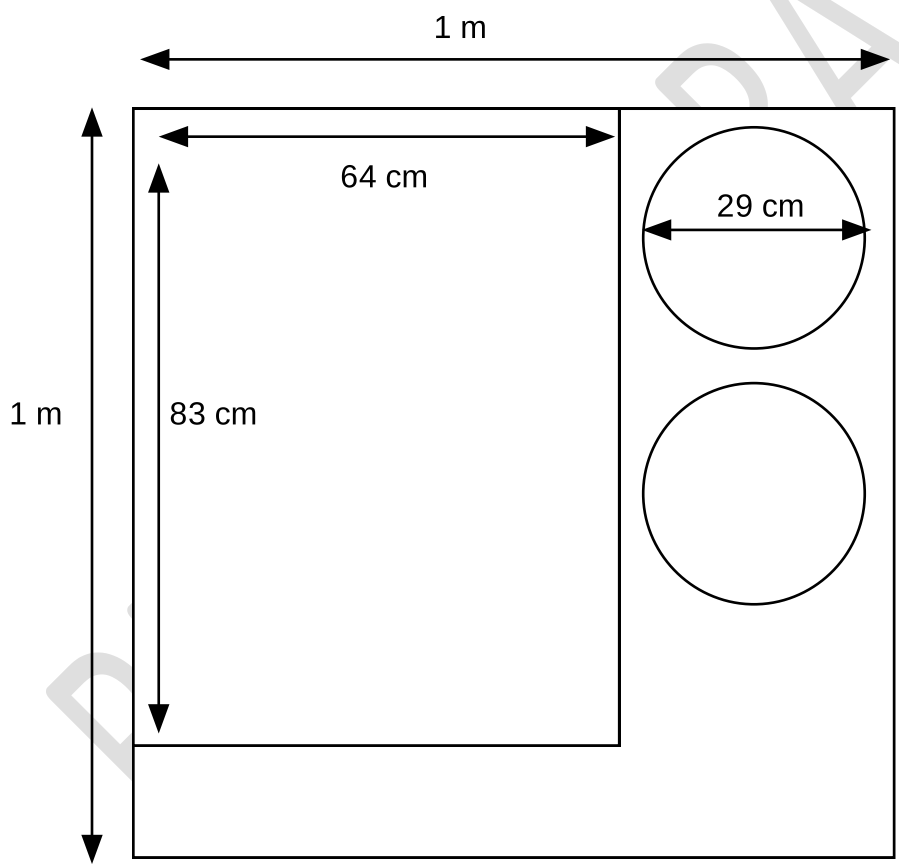
Part 2 of the task is interesting and requires students to be able to visualize the possibilities to solve this problem. Starting with the rectangle in the top corner of the roll they can orient it in two ways. This is the key decision – which way around should it go. With the longer edge along the end of the roll of cloth there is a wasted strip along the edge and the length of the total piece will be equivalent to 64 + 29. Here the student could need to see that both circles can fit within the width of the cloth. However, if the short side (64cm) of the rectangle runs along the end of the roll there is room for the two circles alongside the rectangle. In this case the length of cloth is 82.5cm



This task demands students work across a range of mathematical practices.:

- ***Make sense of problems and persevere in solving them*** **(**P1).
- ***Model*** *(P4) a situation with mathematical representations.*
- ***Use appropriate tools strategically*** (P5), in this case to calculate the circumference of the circle.
- ***Make use of mathematical structure*** (P6).

Rubric Elements

	Sports Bag	Points
1.	Circumference of circular ends is one dimension of main body: C = πd = π x 25 = **78.5** cm Main body is a rectangle measurements 60 + 4 by 78.5 + 4 = **64** by **82.5** cm Two circular ends have diameter **29** cm	
2.	Draws sketch showing that 1 meter of cloth will make the bag. 	
	Total	

Note:

See scoring comments on the “Ms. Olsen’s Sidewalk” task, above

CR 4: Baseball Jerseys

Bill is going to order new jerseys for his baseball team.

The jerseys will have the team logo printed on the front.

Bill asks 2 local companies to give him a price.

1. 'Print It' will charge \$21.50 each for the jerseys.

   Using *n* for the number of jerseys ordered and *c* for the total cost in dollars, write an equation to show the total cost of jerseys from 'Print It'.

   ____

2. 'Top Print' has a Set-Up cost of \$70 and then charges \$18 for each jersey.

   Using *n* to stand for the number of jerseys ordered and *c* for the total cost in dollars, write an equation to show the total cost of jerseys from 'Top Print'.

   ____

3. Use the two equations from questions 1 and 2 to figure out how many jerseys Bill would need to order for the price from 'Top Print' to be less than from 'Print It'.

   Explain how you figured it out.

   ____

____

____

____

____

4. Bill decides to order 30 jerseys from 'Top Print'.
   How much more would the jerseys have cost if he had bought them from 'Print It'?
   Show all your calculations.

   ____

Discussion

Baseball Jerseys addresses:

Content Standards 7.EE: 4, 7.NS:3, 8.EE:8, 8.F.4
Practices P1, P4, and P7
Claims 1 and 4.

Baseball Jerseys
This task considers the costing models of two print companies, one with higher unit costs and the other with lower unit cost but a higher set-up charge. The first part of the task asks students to construct two equations for the cost of each company. The variables *n* and *c* are given and students should be able to produce the two equations $c = 21.5n$ and $c = 70 + 18n$.

The more challenging part of the task comes in question 3.

Here students might construct the inequality [$70 + 18n < 21.5n$] and solve for n. Care would need to be taken to construct the initial inequality correctly but it is then fairly straightforward to solve. Alternatively, students might explore this problem by trying out various values of *n* in order to get a feel for the problem. Although this approach is less elegant it is a more concrete way of tackling this part of the task. Another way of approaching this task would be to look at the per-item cost difference of $3.50 and relate this to the set up cost of $70. Twenty jerseys would balance these two costs, and so on. Or, students might draw the graphs of the two linear functions.

The final section of the task asks the students to find the extra cost increase of buying 30 jerseys from the more expensive supplier ‘Print It’. Assuming that their equations from the first part of the task were accurate this part of the task is a relatively straightforward number problem.
This task demands that students work across a range of mathematical practices. In particular, they need to:

- ***Make sense of problems and persevere in solving them*** **(**P1), particularly in the middle part of the task. Here students need to make choices about their strategy and “plan a solution pathway”.
- ***Look for and make use of structure*** *(P7) in that understanding the properties of linear growth leads one to a solution of the problem.*
- ***Modeling*** *(P4) is involved to a lesser degree, because the student is instructed to construct equations.*

Rubric Elements

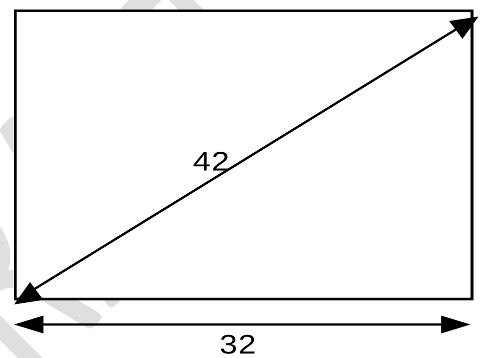
Baseball Jerseys	
	points
1. Gives correct answers: **c = 21.5n**	
2. Gives correct answers: **c = 18n + 70**	
3. Gives correct answer: **21** or **more than 20** Partial credit: 20 Gives a correct explanation such as: The costs will be equal when 21.5n = 18n + 70, 3.5n = 70, n = 20. So it will be cheaper for more than 20 jerseys.	
4. Gives correct answer: **$35** Shows correct work such as: 21.5x 30 = 645 (18 x 30) + 70 = 610 645 – 610 = 35	
Total Points	

Note:

See scoring comments on the "Ms. Olsen's Sidewalk" task, above

CR 5: Jane's T.V.

Jane is hoping to buy a large new television for her den, but she isn't sure what size screen will be suitable for her wall. This is because television screens are measured by their diagonal line.



This 42-inch screen measures 32 inches along the base.

1. What is the height of the screen? ___________

   Show how you know.

2. What is the area of the screen? ______________square inches

3. Jane would like to have a screen 40 inches wide and 32 inches high.

   About what screen size will she need to buy? (Remember that the screen size is measured by length of the diagonal.) ____________inches

   Show how you figured this out.

Discussion

Jane's TV addresses:

Content Standard 8.G.7
Practice P5
Claim 1.

This task is about applying the Pythagorean Theorem to a problem in the context of television sizing.

This first part of the task requires students to recognize that they have been given the hypotenuse of the triangle and so they must apply the theorem carefully. One way of checking this will be that the height will definitely be less than 42 inches, and because of the orientation of the rectangular screen, it should be less than 32 inches. Any student getting 52.8 inches from a misapplication of the theorem should know straight away that they have made an error.

Part 2 of the task simply asks them to use the height measurement to calculate the area of the screen and this is relatively easy to calculate.

Part 3 of the task gives the student the width and height dimensions of a desired screen and asks them to calculate the approximate size (i.e. diagonal) of the screen. This part is similar to part one but applying the theorem in the more straightforward way.

These applications of the Pythagorean Theorem will require the use of a calculator and presents easy opportunities for errors. For this reason good students will have a clear sense, not necessarily as formal as an approximation, of the result and will automatically check their solution if it is not about right. Students will need to be able to use a calculating device properly ensuring that the order of operations is correct.

This task demands students work across a range of mathematical practices. In particular, they need to:

- ***Use appropriate tools strategically*** (P5)

Rubric Elements

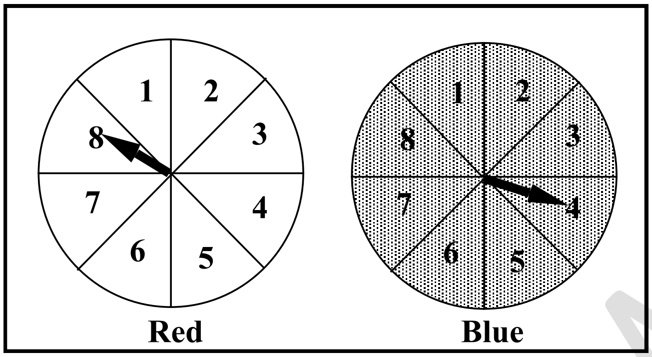
Jane's T.V.	
	points
1. Gives correct answer: Height of screen is **27.2** inches accept √740 Shows work such as: $42^2 = 32^2 + h^2$ 1764 = 1024 + height squared Height squared = 740 Height = 27.2 inches approx 27 inches All correct working. *Partially correct work.*	
2. Gives correct answer: 32 x 27.2 = 870.4 square inches	
3. Gives correct answer**:** 51 inches Shows work such as: $S^2 = 40^2 + 32^2 = 1600 + 1024 = 2624$ S = square root of 2624 = 51.2 approximately 51 inches	
Total Points	

Note:

See scoring comments on the "Ms. Olsen's Sidewalk" task, above

CR 6: The Spinner Game

Sally has made a Spinner game for her class.



Write down 9 **different** numbers on your card.
I will spin both spinners and add up the two numbers I get. If you have that total on your card, you cross it off. The first person to cross off all the numbers wins the prize.

Here are three Spinner Game cards the players made.

Card A

4	5	6
8	9	11
12	13	15

Card B

1	4	6
7	10	12
14	15	17

Card C

2	3	4
5	10	13
14	15	16

This is how the game works. If Sally spins both spinners, and the numbers she gets are 5 and 7, then the total is 12 and the players' cards look like this:

Card A

4	5	6
8	9	11
~~12~~	13	15

Card B

1	4	6
7	10	~~12~~
14	15	17

Card C

2	3	4
5	10	13
14	15	16

Sally keeps spinning until someone’s card has all its numbers crossed off.

1. Which card has the best possible chance of winning?
   Give reasons for your answer.

2. Fill in a card that has the best chance of winning.

3. Explain how you chose the numbers for your card.

Discussion

The Spinner Game addresses:

Content Standard 7.SP:8
Practices P1, P2, P3, P7, P8
Claims 1, 2, 3 and 4.

This challenging task requires students to reason with probability. Part 1 of the task asks the students to explain their answer to the question "which card has the best chance of winning?"

To make progress on this task, students must understand the idea of probability space. They may have encountered this idea in the context of two coins being tossed or the rolling of a pair of dice. The analogy here is that the most common total is going to be 9, for which there are 8 possible combinations on the spinners. The probabilities reduce either side of this, and are symmetrical in the sense that 8 and 10, 7 and 11, etc. are equally probable (and each set less than the preceding one).

What should be immediately clear to the student is that 1 is not possible from these two spinners so Card B cannot be a solution to the question. How to differentiate between Card A and Card C will not be so clear to many students. The key idea is that *the more ways you can make a sum from the numbers on the two spinners, the more likely it is that that sum will come up.* If they have this sense of the higher probability of totals at or around 9 it becomes clear that Card C has fewer of these numbers and more from the extremes of the range.

Part 2 of the question asks the student to fill in the card that has the best chance of winning. Students will need to have a clear sense of the structure of this space. If they do, it is clear (as discussed above) that 9 is the most probably outcome, 8 and 10 the two next most probable outcomes, and so on - so that a card with the number 5, 6, 7, 8, 9, 10, 11, 12, and 13 is the best bet. A student's explanation might be somewhat informal, based upon the idea that numbers around 9 are most likely to come up, or it might use the probability space to formalize this argument. Either way there is a requirement to demonstrate high quality reasoning to develop their argument.

This task demands that students work across a range of mathematical practices. In particular, they need to:

- ***Construct viable arguments and critique the reasoning of others*** (P3).
- ***Make sense of problems and persevere in solving them*** (P1). Students will need to explore the problem and develop some strategy for approaching it. This will include choosing appropriate mathematics.
- ***Reason abstractly and quantitatively*** (P2), in particular through moving from the probability game context to an abstracted probability space diagram.
- ***Look for and make use of mathematical structure*** (P7).
- ***Look for and make use of regularity in reasoning and argument*** (P8).

Rubric Elements

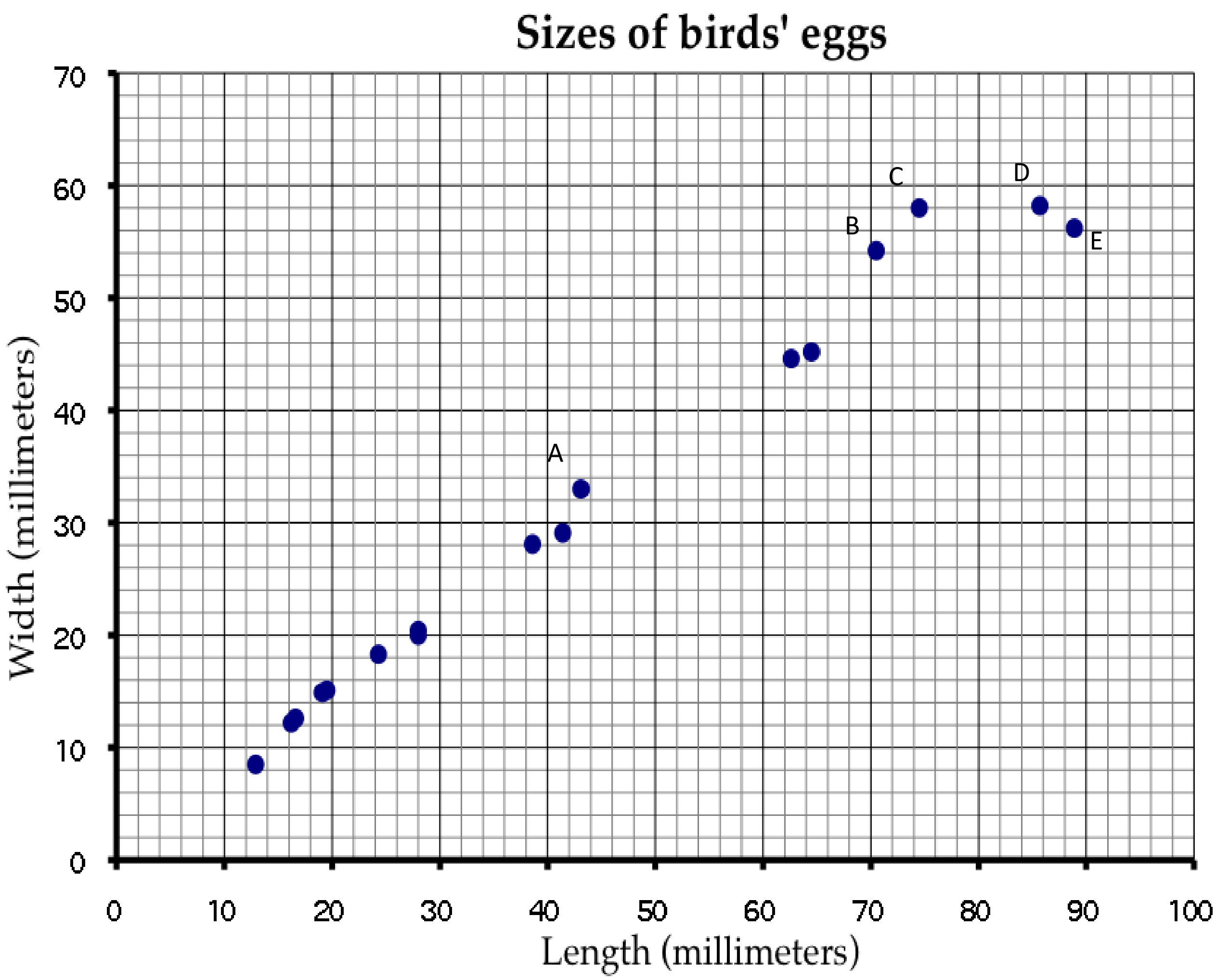
	The Spinner Game	Points
1.	Shows some evidence of working out probabilities or possible scores on diagram or listing. Complete listing or lattice diagram or distribution of scores. States that: Card B: 1 and 17 are impossible, so this card cannot win. Card C: contains extreme/unlikely numbers because they have few combinations. Card A: contains middle/more likely numbers because they have more combinations. Compares cards: Card A is the most likely to win.	
2.	Chooses numbers in the range 2 to 16. Chooses numbers in the range 5 to 13. States reasons for choice	
	Total Points	

Note:

See scoring comments on the "Ms. Olsen's Sidewalk" task, above

CR 7: Bird and Dinosaur Eggs

This scatter diagram shows the lengths and the widths of the eggs of some American birds.



1. A biologist measured a sample of one hundred Mallard duck eggs and found they had an average length of 57.8 millimeters and average width 41.6 millimeters.

   Use a **X** to mark a point that represents this on the scatter diagram.

2. What does the graph show about the connection between the lengths of birds' eggs and their widths?

____

____

____

3. People think dinosaurs laid huge eggs - but they didn't! Fossils show that sauropods, which could weigh as much as 20 tons, actually grew from eggs that were only 180 millimeters long. If sauropod eggs were the same shape as bird eggs, approximately how wide would they be?

____

4. Duckbill dinosaurs (hadrosaurs) could grow to 10 - 15 meters long. Their eggs were 10-12 cm long and 7.9 cm wide. Based on these numbers, would you argue that duckbill eggs were:

   a. thinner
   b. about the same ratio
   c. rounder

   than bird eggs? Explain.

____

____

____

Discussion

Bird and Dinosaur Eggs addresses:

Content Standard 8.EE.5, 8.SP.1
Practices P2, P5, P6
Claims 1 and 4.

In this task students engage in the interpretation of data on the sizes of bird and dinosaur eggs.

The task starts with a scatter diagram recording the length and width of the eggs. Students are asked to plot a point on the diagram representing a newly measured egg. This involved the accurate use of scales and rounding of decimal measurements to the more approximate scale.

They are then asked to describe the relationship between width and length of the eggs. This involves recognizing that the ratio is essentially linear, and that the relationship can be used for prediction.

When looking at graphs like this, students can be asked to *read the data, read between the data* and *read beyond the data*. Part 3 asks students to extrapolate beyond the given data, and part 4 asks them to decide whether variable data fit, more or less, the linear trend described in the graph. (The ratios in part 4 range from about 1.26 to 1.52, which the slope of the approximation line is roughly 1.33 - so one might argue that the duckbill eggs were at least a bit rounder.

This task demands that students work across a range of mathematical practices. In particular, they need to:

- ***Reason abstractly and quantitatively*** (P2), moving between the abstracted graphical representation and what they mean.
- ***Use tools strategically*** (P5) in extrapolating beyond the given table.

Attend to precision (P6) in comparing ratios.

Rubric Elements

	Points
1. Places point correctly on graph. Accept points within 1 square of correct position.	
2. Gives a correct description such as: Generally, the greater the length of the egg, the greater is its width.	
3. Gives correct answer: **126** mm approximately. Accept values between 115 and 135. Gives a correct explanation such as: "I multiplied by about .7" or "The width is about 2/3 (or ¾) of the length" or "I doubled the width of the egg that is 90 mm long"	
4. Gives a correct answer (either "it's close" or "maybe slightly rounder" and justifies the answer correctly, either by plotting or computing the relevant ratios.	
Total Points	

Note:

See scoring comments on the "Ms. Olsen's Sidewalk" task, above

CR 8: Taxi Cabs

Max is organizing a trip to the airport for a party of 75 people.

He can use two types of taxi.

A small taxi costs $40 for the trip and holds up to 4 people.

A large taxi costs $63 for the trip and holds up to 7 people.

1. a. If Max orders 6 large taxis, how many small taxis will he need? ____________________

   b. How much will the total cost be? ___________________

Max can organize the journey more cheaply than this!

How many taxis of each type should Max order, to keep the total cost as low as possible? Explain.

____

Discussion

Taxi Cab addresses:

Content Standard 8.EE.8
Practices P1, P2, P4
Claims 1, 2, 3, and 4.

Problem 1 provides some simple scaffolding to help insure that students understand the context. Max orders 6 large taxis and needs to know how many small taxis he will need. The first stage of the approach to this question is pretty clear: 6 large taxis hold *42* people, which leaves a further 33 to be taxied. The difficulty here is when students take these remaining 33 passengers and divide by 4, the number of passengers per taxi. The resulting decimal leaves the student with a common rounding problem: is it 8 or 9 taxis? Once the student has resolved this problem (8 taxis only hold 32 so it needs to be 9) the follow on question regarding the cost is relatively easy.

Then the task opens up considerably, asking the student to minimize the cost of the journey. Given the scaffolding above, the student might well choose to vary the number of large taxis and see what happens to the total cost. But there is still a lot of work to be done – there are often spare places in the last taxi so this situation does not behave quite as neatly as many mathematical problems.

One reason to vary the number of large taxis is that the cost using small taxis is $10 per person (when the taxi is full) whereas the large taxi is $9. This suggests that using as many large taxis as possible would be a good strategy. But, the "empty seat" problem means that some cases have to be worked out:

Large taxis	People in large taxis	People in small taxis	Small taxis needed	Cost of large taxis	Cost of small taxis	Total cost
11	all 75	0	0	693	0	693
10	70	5	2	630	80	710
9	63	12	3	567	120	687
8	56	19	5	504	200	704
7	49	26	7	441	280	721

This is a good example of how a task, particularly one that works well from a realistic context, can provide both surprises and rich opportunities for mathematical modeling and reasoning..
This task demands that students work across a range of mathematical practices. In particular, they need to:

- ***Make sense of problems and persevere in solving them*** (P1), particularly the second part of the problem.
- ***Reason abstractly and quantitatively*** (P2), decontextualising and recontextualising between the situation and the mathematics.
- ***Model with mathematics*** (P4).

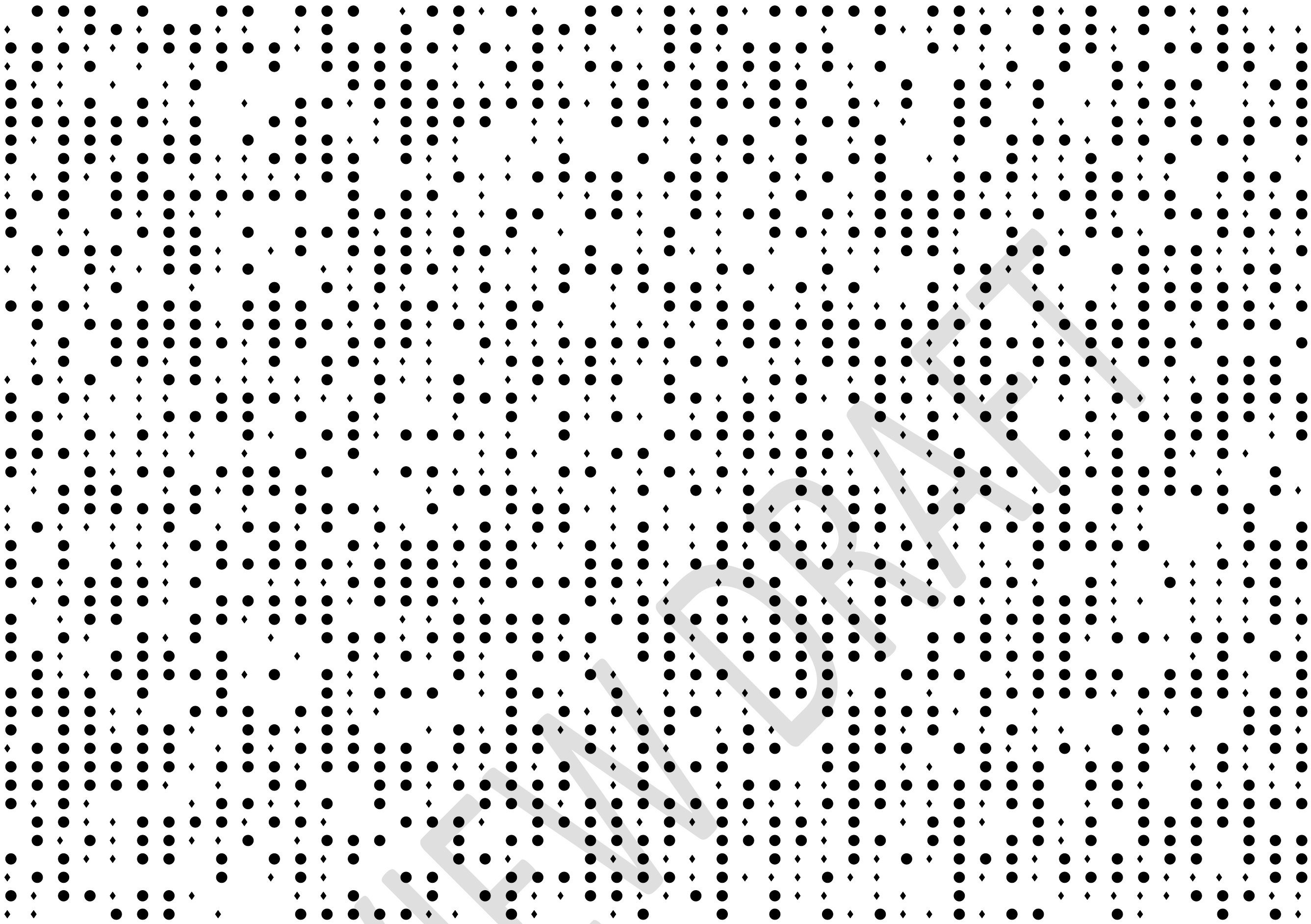
Rubric Elements

	Taxi Cabs	Points
1. a	6 large taxis hold 42 people 75 - 42 = 33 people 33 people need 9 small taxis with 3 empty seats 6 large taxis cost 6 x $63 = $378 9 small taxis cost 9 x $40 = $360 Total cost **$738**	
2.	The best strategy is to increase the number of large taxis (because each seat costs $9) and decrease the number of empty seats in the small taxis. Large taxis / Small taxis / Cost in $ 6 / 9 / 738 7 / 7 / 721 8 / 5 / 704 9 / 3 / 687 **no empty seats** 10 / 2 / 710 **$687 is the lowest possible cost**	
	Total Points	

Note:

See scoring comments on the "Ms. Olsen's Sidewalk" task, above

CR 9: Counting Trees



This diagram shows some trees in a plantation.

The circles ● show old trees and the triangles ♦ show young trees.

Tom wants to know how many trees there are of each type, but says it would take too long counting them all, one-by-one.

1. What method could he use to estimate the number of trees of each type?
   Explain your method fully.

___

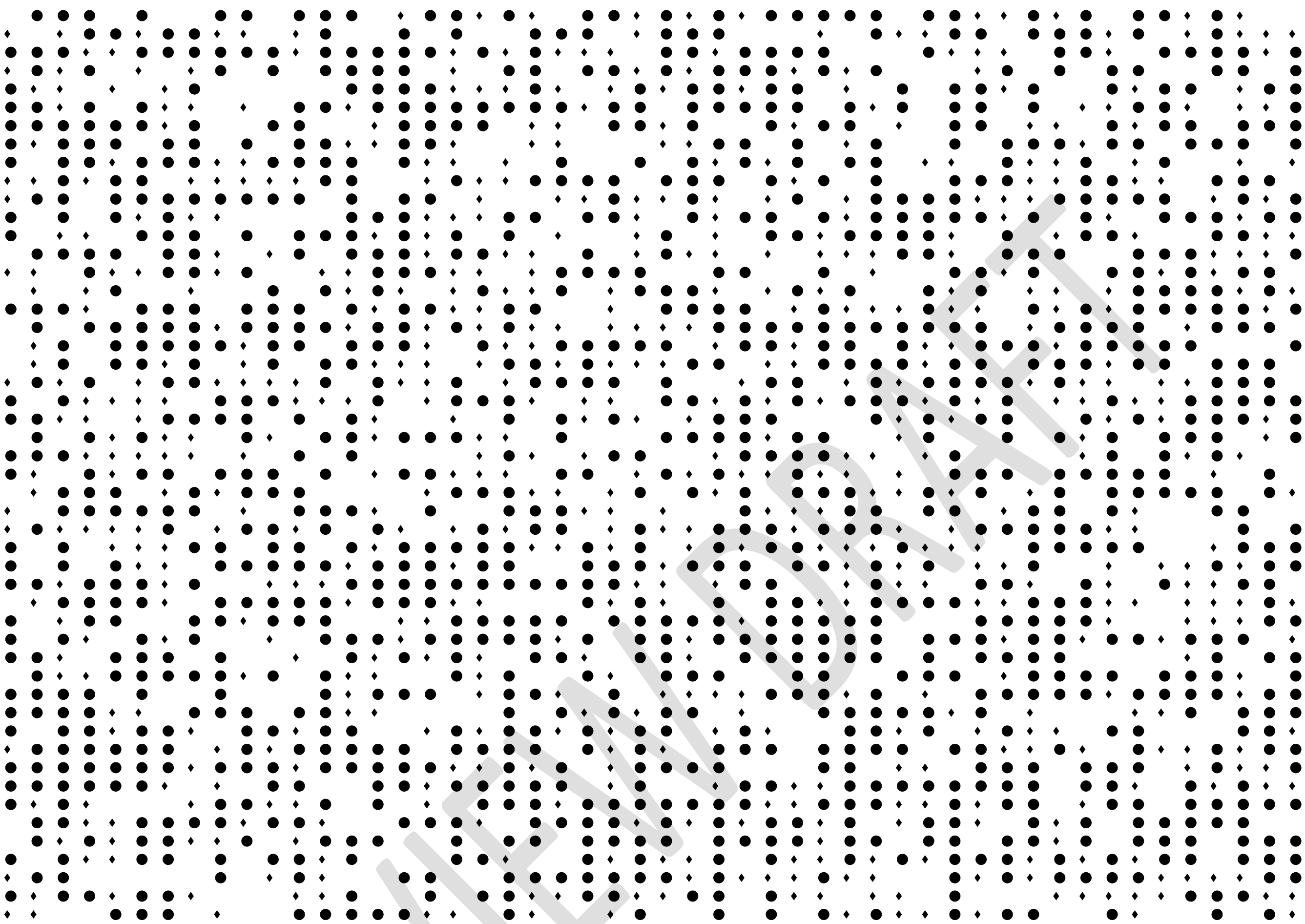
___

___

2. Use your method to estimate the number of:

   (a) Old trees (b) Young trees

Counting Trees WORKSHEET



Discussion

Counting Trees addresses:

Content Standard 7.RP
Practices P1, P5, and P6
Claims 1, 2, and 4.

In this task students use ratios to calculate approximate solutions. They need to make decisions about how to tackle the problem and decide how much information is needed to increase the accuracy of the approximation.

They are asked to choose a method for estimating the numbers of different types of trees in a large plantation. One simple approach, given that the trees are arranged in a grid is to count the numbers of each tree in one row and then multiply by the number of rows. This approach would not work if the arrangement was more random. In that case a smaller area could be sectioned off. The area as a proportion could be estimated and the necessary calculations made. This approach would also work with this problem. The student might count the number of each type of tree in more than one row in order to increase confidence in the estimates.

This task demands students work across a range of mathematical practices. In particular, they need to:

- ***Make sense of problems and persevere in solving them*** (P1).
- ***Use appropriate tools strategically*** (P5).
- ***Attend to precision*** (P6).

Rubric Elements

	Counting Trees	Points
1.	Explains that a small representative section could be selected. Then the number of old trees in that section could be counted The number of young trees in that section could be counted. These numbers could be used to make an estimate for the whole area. *Partial credit* For a partially correct explanation.	
2.	Accept different organized sectioning methods. For example: The total area is 17.5 x 12 sq cm For example if we select an area 2cm x 2cm. Counting the number of old trees, we get 28 Counting the number of young trees, we get 11. An estimate of the number of old trees is 28 x 17,5 x 12 ÷ 4 = 1470 approximately **1500**. Accept values in the range 1200 to 1600 An estimate of the number of young trees is 11 x 17,5 x 12 ÷ 4 = 577 approximately **600**. Accept values in the range 500 to 700	
	Total Points	

Note:

See scoring comments on the “Ms. Olsen’s Sidewalk” task, above

CR 10: Shelves

Pete is making a bookcase. He has plenty of bricks and can get planks of wood for $2.50 each.

Each plank of wood measures 1 inch by 9 inches by 48 inches. Each brick measures 3 inches by 4.5 inches by 9 inches.

For each shelf, Pete will put three bricks at each end then put a plank of wood on top. The diagram shows three shelves.

Pete wants five shelves in his bookcase.

1. How many planks of wood does he need? ________________________

2. How many bricks does he need? ________________________

3. How high will the shelves be? ________________________

4. How much will the bookcase cost? ________________________

5. If he makes a bookcase that has n shelves, how high will the bookcase be?

____

Discussion

Shelves addresses:

Content Standards 8.F.1, 8.F.2
Practices P1 and P7
Claim 4.

This is a simple modeling problem, which calls for keeping track of the various quantities and their dimensions and costs. For each of subparts 1 through 4, the student must decide which information is relevant. Part 5 calls for abstracting some o0f the computations done in parts 1 through 4.

This task demands that students:

- ***Make sense of problems and persevere in solving them*** (P1).
- ***Look for and make use of structure*** (P7).

Rubric Elements

Shelves	
	points
1. Gives correct answer: **5** 2. Gives correct answer: **30** 3. Gives correct answer: **50** inches 4. Gives correct answer: **$12.50** 5. Gives correct answer: H = 10n inches	
Total Points	

Note:

See scoring comments on the “Ms. Olsen’s Sidewalk” task, above

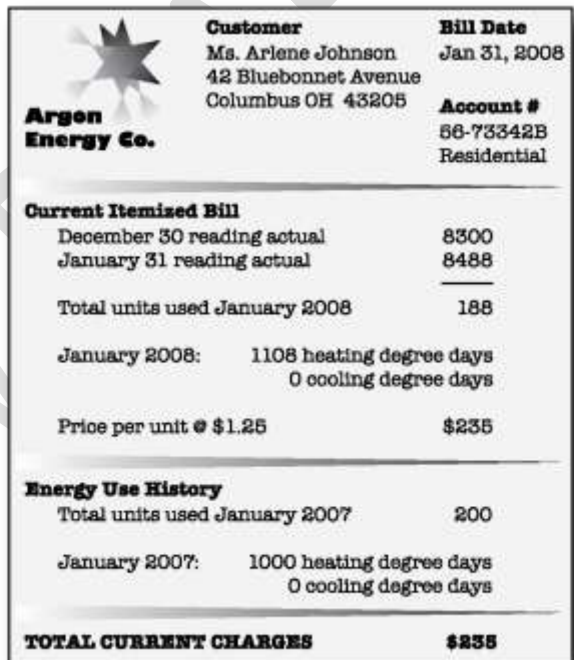
Part III: An Extended Performance Task

Gas Bills, Heating Degree Days, and Energy Efficiency

Here is a typical story about an Ohio family concerned with saving money and energy by better insulating their house.

Kevin and Shana Johnson's mother was surprised by some very high gas heating bills during the winter months of 2007. To improve the energy efficiency of her house, Ms. Johnson found a contractor who installed new insulation and sealed some of her windows. He charged her $600 for this work and told her he was pretty sure that her gas bills would go down by "at least 10 percent each year." Since she had spent nearly $1,500 to keep her house warm the previous winter, she expected her investment would conserve enough energy to save at least $150 each winter (10% of $1,500) on her gas bills.

Ms. Johnson's gas bill in January 2007 was $240. When she got the bill for January 2008, she was stunned that the new bill was $235. If the new insulation was going to save only $5 each month, it was going to take a very long time to earn back the $600 she had spent. So she called the insulation contractor to see if he had an explanation for what might have gone wrong. The contractor pointed out that the month of January had been very cold this year *and* that the rates had gone up from last year. He said her bill was probably at least 10% less than it would have been without the new insulation and window sealing.



Argon Energy Co.

Customer	Bill Date
Ms. Arlene Johnson 42 Bluebonnet Avenue Columbus OH 43205	Jan 31, 2008
	Account # 56-73342B Residential

Current Itemized Bill

December 30 reading actual	8300
January 31 reading actual	8488
Total units used January 2008	188
January 2008: 1108 heating degree days 0 cooling degree days	
Price per unit @ $1.25	$235

Energy Use History

Total units used January 2007	200
January 2007: 1000 heating degree days 0 cooling degree days	

TOTAL CURRENT CHARGES	$235

Ms. Johnson compared her January bill from 2008 to her January bill from 2007. She found out that she had used 200 units of heat in January of 2007 and was charged $1.20 per unit (total = $240). In 2008, she had used 188 units of heat but was charged $1.25 per unit (total = $235) because gas prices were higher in 2008. She found out the average temperature in Ohio in January 2007 had been 32.9 degrees, and in January of 2008, the average temperature was more than 4 degrees colder, 28.7 degrees. Ms. Johnson realized she was doing well to have used less energy (188 units versus 200 units), especially in a month when it had been colder than the previous year.

Since she used gas for heating only, Ms. Johnson wanted a better estimate of the savings due to the additional insulation and window sealing. She asked Kevin and Shana to look into whether the "heating degree days" listed on the bill might provide some insight.

Winter Temperatures and "Heating Degree Days"

Kevin and Shana quickly found a description of "degree days" on Wikipedia at http://en.wikipedia.org/wiki/Heating_degree_day. Here is some of what they learned:

Degree Days are a method for determining cumulative temperatures over the course of a season. They were originally designed to evaluate energy demand and consumption, and are based on how far the average temperature departs from a human comfort level of 65°F. Each degree of temperature above 65°F is counted as one cooling degree day, and each degree of temperature below 65°F is counted as one heating degree day. For example, a day with an average temperature of 45°F is counted as having 20 heating degree days. The number of degree days accumulated in a day is proportional to the amount of heating/cooling you would have to do to a building to reach the human comfort level of 65°F. The degree days are accumulated each day over the course of a heating/cooling season, and can be compared to a long term (multi-year) average, or norm, to see if that season was warmer or cooler than usual.

Task Description

Assess the cost-effectiveness of Ms. Johnson's new insulation and window sealing. In your assessment, you must do the following:

- Compare Ms. Johnson's gas bills from January 2007 and January 2008, estimate her savings due to the new insulation and sealing, and explain your reasoning.
- Decide whether the insulation and sealing work on Ms Johnson's house was cost-effective, and provide evidence for your decision.

Internet Resources

Heating and Cooling Degree Days - Definitions and Data Sources

Definition and discussion - http://en.wikipedia.org/wiki/Heating_degree_day
Standard for HDDs and CDDs - http://www.weather2000.com/dd_glossary.html
National Climatic Data Center - http://www.ncdc.noaa.gov/oa/documentlibrary/hcs/hcs.html
City-specific data - http://www.degreedays.net (use weather station KOSU for Columbus)

Natural Gas Usage and Natural Gas Prices

U.S. Dept. of Energy - http://www.eia.doe.gov/neic/brochure/oil_gas/rngp/index.html
Ohio Consumers' Council - http://www.pickocc.org/publications/handbook/gas.shtml
Ohio Public Utilities Commission - price comparison chart for Columbia Gas of Ohio - http://www.puco.ohio.gov/Puco/ApplesToApples/NaturalGas.cfm?id=4594

Rubric Elements

Heating Degree Days		Rubric	
	1. Equating a rate between 2007 and 2008 or visa-versa. Converting the amount of heating units used in 2008 by the rate used in 2007 (heating units to heating degree days): 188/1108 = 0.1696 0.1696 • 1000 **= 169.6** heating units (by 2007 standards) or Converting the amount of heating units used in 2007 by the rate used in 2008: (200/1000) = 0.2 0.2 • 1108 **= 221.6** heating units (by 2008 standards)	2 2 3 2 2 3	7
	2. Determines cost of usage in one year compared to the other year. Cost in 2007 rate of 2008 energy usage 169.6•$1.20 = **$203.52** or Cost in 2008 rate of 2007 energy usage 221.6•$1.25 = **$277.00**	2 3 3	5
	3. Determines the percent of savings from one year to next. Based on year 2007: $240.00-$203.52 = $36.48 (savings) $36.48/$240.00 = 15.2% or Based on year 2008: 277.00-235.00 = $42.00 (savings) $42.00/$277.00 = 15.2% *Partial Credit* If the savings was calculated using the wrong difference and/or divisor to determine the percent savings is an incorrect then partial credit.	3 4 3 4 (3)	7
	Special Case (short cut solution) (0.2 – 0.17)/0.20 = 15% savings	s.c. 19	
	4. Concludes that the savings are cost-effective to make the investments. (Based on correct work)	2	2
	5. Determines how long it will take to pay-off the investment such as: $600 investment divided by $42 (per winter month) = 14.2 (winter months). If there are 5 winter months per year, it would take three years to make up her investments.	4	4
	Total		**25**

Notes on This Task

Mathematical themes of this task

- Proportional reasoning
- Interpreting verbal descriptions of mathematical situations
- Using ”heating degree days” to measure energy use
- Constructing and comparing rates and ratios
- Linear modeling
- Determining cost-effectiveness
- Preparing for Calculus
- Exploring efficiency standards

Mathematical Analysis

Proportional reasoning

At the core of this task is the recurring question, ”What is proportional to what?” Answers to this question include:

- The number of heating units is (presumably) proportional to the number of heating degree days.
- The total monthly cost for gas is proportional to the number of heating (or cooling) units used. Note that the rate determining this relationship varies from month to month.
- There is also a linear relationship between temperature and heating degree days.

The questions in the task focus on savings as a percent, so the proportional reasoning involves translating comparisons into percent increases and decreases, as is discussed below. The task also involves determining the conditions under which a 10% savings would occur, which requires using proportional reasoning to ”work backwards.”

Interpreting verbal descriptions

Keeping track of all of the variables involved in the situation requires a strong ability to interpret verbal description in terms of quantitative relationships. In addition to the variables directly involved (bill amount, heating units, temperature, heating degree days), the task refers to fluctuations in the price of gas as a major factor in consumer energy costs, making the number of variables involved in the work realistic for the context. Different approaches to this task depend on different ways of organizing the information provided in order to see what would be a useful comparison between the two months.

Heating degree days

The context of the task teaches students about using ”heating degree days” (the number of degrees below 65 degrees Fahrenheit of an average daily temperature, per day) as a way of measuring temperature that focuses attention on energy usage. Using this unit of measurement invites an exploration of the impact of weather on seasonal heating and cooling needs, and it foregrounds the basic idea that heating and cooling require more energy with more extreme temperatures. Treating the relationship between temperature and energy use as approximately proportional makes the questions in this task reasonable.

Linear modeling

Using heating degree days as the measure of energy use relies on a linear model of energy use in that it assumes energy use changes at a constant rate relative to temperature. For example, energy use on a day with an average temperature of 55 degrees (10 heating degree days) is assumed to be half that for a day with an average temperature of 45 degrees (20 heating degree days). The validity of such a model can be explored using the additional information in the tables provided in the task.

Constructing and comparing rates and ratios

There are several ways to set up ratios and rates for comparison in this task. One is to start with the ratio between the number of heating degree days in each month (1108/1000 = 1.108), which indicates that January 2008 was 10.8% colder than January 2007. This suggests, according to the linear model just mentioned, that Ms. Johnson would have used 10.8% more energy in January 2008 than she did in January 2007, that is, 1.108 • 200 = 221.6 units, if she hadn't had the insulation and window sealing work done. From this, we can see that her energy use was approximately 15.2% less than it would have been without the energy efficiency measures, since 221.6/188 ≈ 0.848.

Assuming energy use is proportional to temperature, and accounting for the increase in price per unit of heat, Ms. Johnson's bill in January 2008 without the energy efficiency measures would have been: 221.6 units of heat • $1.25/unit of heat = $277. Her actual bill was $235, so her savings was $42 (i.e., approximately 15.2%).

Some students may skip the conversion from percent increase in heating degree days into units of heat used, and jump directly to the ratio of units used in each month: 188/200 = 0.94, indicating her energy use was 6% less in January 2008 than it was in January 2007. This would then suggest a savings of 16.8% (10.8% + 6% = 16.8%), rather than 15.2% (without accounting for the price increase). The answer 16.8% is incorrect because it combines percent change in temperature (heating degree days) with percent change in energy use, as if these were equivalent quantities. This presents an opportunity to explore proportionality vs. equivalence, and students should be allowed to grapple on their own with this issue as much as possible.

Another approach is to begin with the rate of energy use per heating degree day for each month:

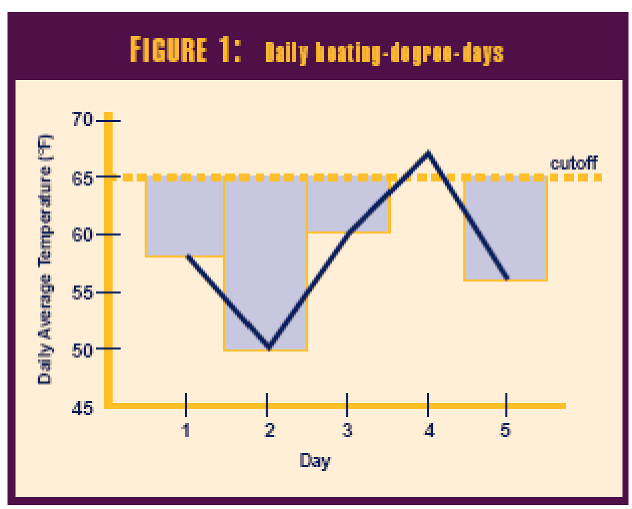
January 2007: 200/1000 = 0.2 units of heat per heating degree day

January 2008: 188/1108 ≈ 0.16968 units of heat per heating degree day

This indicates an increase in energy efficiency of approximately 15.2% (0.16968/0.2 ≈ 0.848), as before.

Understanding cost-effectiveness

The task also requires an understanding of what is meant by "cost-effective" and an ability to determine what would count as cost-effective. This requires a basic understanding of distributed cost over time and of short-term vs. long-term investment and savings. Understanding why the energy efficiency work done to Ms. Johnson's home was cost-effective provides a basis for explaining how to assess cost-effectiveness more generally.



Preparing for Calculus

In dealing with accumulation of heating degree days, the task also offers opportunities to foreshadow some of the basic ideas in Calculus. Specifically, the total number of heating degree days accumulates over time within a given period as average temperature fluctuates over that period, and the accumulation is similar to integration in Calculus. The graph at right illustrates the idea, with each bar representing the number of heating degree days (the number of degrees below 65 degrees) for each day in a five-day period, and the line showing average temperature for each day. The sum of the areas of the bars is the total number of heating degree days accumulated during the period.

Efficiency standards

Finally, the task also includes an opportunity to explore the mathematics of efficiency standards for appliances, vehicles, and buildings (e.g., the "Energy Star" rating system for appliances, fuel efficiency and emissions standards for vehicles, and the Green Building Council's energy efficiency standards).

Source: Mathematical Sciences Education Board, National Research Council. *High School Mathematics at Work: Essays and Examples for the Education of All Students*. (Washington, D.C.: National Academy Press, 1998, p. 55)

Problem Sources

Part I: Short items

1: MARS
2: MARS
3: SBAC
4: MARS
5: PISA
6: MARS
7: PISA
8: MARS
9: MARS
10: MARS
11: SBAC
12: SBAC
13: SBAC

Part II: Selected Response Tasks

CR 1: SBAC
CR 2: MARS
CR 3: MARS
CR 4: MARS
CR 5: MARS
CR 6: MARS
CR 7: MARS
CR 8: MARS
CR 9: MARS
CR 10: MARS

Part III: Extended Performance Task

Ohio Department of Education and the Stanford University School Redesign Network